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# LEVELING;

## BAROMETRIC, TRIGONOMETRIC AND SPIRIT.

BY


IRA O. BAKER, C. E.,

*Professor of Civil Engineering,*
*University of Illinois.*


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# PREFACE.

The following pages were prepared as a part of the author's lectures on Geodesy, given to succeeding classes in the University of Illinois, and are now published for the greater convenience of his students, and with the hope that they may be useful to others. The author does not claim that there is anything new or original in this volume; he has simply condensed into a single book what heretofore could be found only scattered through many. The object was to give all that was necessary for a thorough comprehension of the principles involved and an intelligent understanding of the method of applying them. Acknowledgments have been made as far as the source of information was known, but as the lectures were prepared during odds and ends of time, and modified

from year to year, it is possible that proper credit has not been given in all cases. The demonstrations of the formula are original, but the resulting formula is left in that form which the best authorities consider most desirable; the authority for the formula is always given. The attempt has been made to point out all the sources of error, and to give data showing the degree of accuracy attainable by each method.

The author trusts that this little volume will be found to contain all of the essential principles and facts respecting those subjects of which it treats, and hopes that it may be serviceable to those who desire an understanding of those subjects.

University of Illinois,
Nov. 6, 1886.

# LEVELING;

### *BAROMETRIC, TRIGONOMETRIC, AND SPIRIT.*

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§ 1. Hypsometry is that branch of geodesy which treats of the measurement of heights, either absolute when referred to the sea-level, or relative between any two points on the earth's surface. There are three principal and independent methods in use.

The first depends upon the law of the decrease of pressure of the atmosphere with an increase of altitude. This method employs the barometer, and may be called barometric leveling. The second depends upon the measurement of the vertical angle and the horizontal distance. It employs an angle instrument, the horizontal distance usually being given by triangulation; the elevation is then determined from the known parts of a triangle, hence the name trigono-

metric leveling. The third consists in measuring the distance of two points above or below a horizontal line. This is ordinary leveling, in which a leveling instrument gives a visual horizontal line. Notice that the second is the only one applicable when one or both stations is inaccessible. These three methods will be treated separately in succeeding chapters.

§ 2. In a geodesic survey conducted to determine the size and figure of the earth, the vertical element is required, although it is not nearly as important as the horizontal. For example, the profile of the base must be determined so that the measurement may be reduced to a level line, and its elevation above the sea-level must be known, that the measurement may be reduced to the level of the sea; in planning the triangulation, at least the approximate difference of level of the vertices of the triangles is required to determine the height to which the signals must be elevated that they may be visible from the other stations.

When the object of the survey is a map, the vertical element is more important; if the map is to serve as a basis of a geological or topographical survey, the vertical element is equally as important as the horizontal element, or perhaps more so. If the map is to be useful in the preliminary examination for railroads, canals, river improvements, etc., the vertical element becomes the most important.

§ 3. Of the three co-ordinates necessary to completely determine a point—1, vertical distance; 2, horizontal distance; and 3, direction—there is the greatest uncertainty in the results for the vertical distance. It is only very recently that leveling has been done with an accuracy that would compare favorably with other geodesic operations. This is partly due to the fact that early geodesic operations were carried on for scientific objects which did not involve the vertical element, and partly to the natural difficulties, which will be discussed presently.

## Chapter I.

### LEVELING WITH MERCURIAL BAROMETER.

§ 4. *Barometric Leveling in General.*—The difference in height of two places may be determined by finding the difference of their depths below the top of the atmosphere. The height of the atmosphere above any point is determined by weighing it. This is done by trying how high a column of mercury or other liquid the column of air above it will balance, or by finding the pressure it will exert against an elastic box containing a vacuum, or by observing the temperature at which a liquid boils, *i.e.*, by observing the temperature at which the pressure of the atmosphere just balances the tension of the vapor. This gives rise to three slightly different methods, according to whether the instrument is a mercurial barometer, an aneroid, or a thermo-barometer or boiling-point apparatus.

Barometric leveling is specially adapted to finding the difference of level between

points at considerable horizontal or vertical distance apart. Under these conditions it is the most speedy, but the least accurate, of any of the methods of leveling. It is very valuable in making geographical surveys of large areas for determining the elevation of stations to be occupied by the topographer. It is also well suited to making a reconnoissance for a railroad, or for a scheme of triangulation.

### ART. 1. THE INSTRUMENT.

§ 5. *Description.*—There are two kinds of mercurial barometers, the cistern and the syphon; the former is the best and most reliable for hypsometrical purposes. The general form of the cistern barometer needs no description; Fig. 1 shows the cistern and details at the lower end of the instrument. "The cistern is made up of a glass cylinder F, which allows the surface of the mercury to be seen, and a top plate G, through the neck of which the barometer tube *t* passes, and to which it is fastened by a piece of kid leather, making a strong but flexible joint.

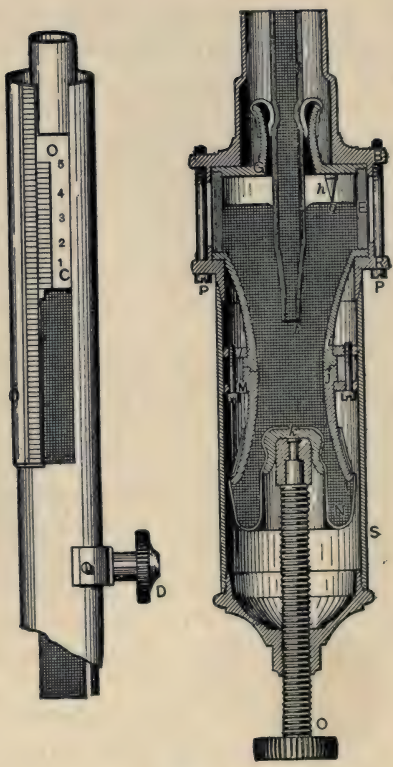


Fig. 1.

To this plate, also, is attached a small ivory point *h*, the extremity of which marks the commencement or zero of the scale above. The lower part, containing the mercury, in which the end of the barometer tube *t* is plunged, is formed of two parts *i j*, held together by four screws and two divided rings *l m*. To the lower piece *j* is fastened the flexible bag N, made of kid leather, furnished in the middle with a socket *k*, which rests on the end of the adjusting screw O. These parts, with the glass cylinder F, are clamped to the flange B by means of four large screws P and the ring R; on the ring R screws the cap S, which covers the lower parts of the cistern, and supports at the end of the adjusting screw O. G, *i*, *j*, and *k* are of boxwood; the other parts of brass or German silver. The screw O serves to adjust the mercury to the ivory point, and also, by raising the bag, so as to completely fill the cistern and tube with mercury, to put the instrument in condition for transportation."*

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* Smithsonian Mis. Col., Vol. I.

§ 6. *Filling the Barometer.*—It is no slight matter to properly fill a barometer. It can best be done by the manufacturer, who has all the facilities; but as it is sometimes necessary for the observer to refill it, the following hints are given. Tubes require refilling owing to the breaking of the glass or to the entrance of a bubble of air.

The mercury should be chemically pure and free from oxide; otherwise it will adhere to the glass and tarnish. Moreover, if it is not pure, the height of the barometric column will not be correct; only mercury should be used which has been purified by distillation. For the best results, the mercury should be boiled in the tube to expel moisture and air; but this can not always be done, and fair results can be obtained without boiling. The following description of the method of filling is from Smithsonian Report, 1859, page 440, and is recommended by Williamson (p. 140).

"The glass tube, which should be clean and dry, must have its open end

ground straight and smooth, so that it can be closed air-tight with the finger, which should be covered with a piece of chamois or kid skin. Warm well both mercury and glass tube, and filter in through a clean paper funnel with a very small hole (about $\frac{1}{50}$ of an inch) below, to within one-fourth of an inch of the top. Shut up the end and turn the tube horizontal, when the mercury will form a bubble that can be made to run from end to end by change of inclination, which will gather all the small air bubbles visible that adhered to the inside of the glass tube during filling. Now let that bubble, which has grown somewhat larger, pass to the open end. Fill up this time with mercury entirely and shut tightly. Then reverse the tube over a basin, when, by slightly relieving the pressure against the end, the weight of the column of mercury will force some out, forming a vacuum above, which ought not to exceed one-half an inch. Closing up again tightly, let this vacuum bubble traverse the length of the tube on

the several sides, when it will absorb those minute portions of air, now greatly expanded from removed atmospheric pressure, that were not drawn at the first gathering. The perfect freedom from air is easily recognized by the sharp concussions with which the column beats against the sealed end, when, with a large vacuum bubble, the horizontally held tube is slightly moved."

"A barometer so prepared will probably read lower by a few thousandths than if the tube had been boiled, but in a stationary barometer its error will probably not soon change, and carrying on horseback will be apt to improve it rather than otherwise, as it is then carried with the cistern uppermost, and the bubbles will be jolted toward the open end." * If possible it should be compared with a standard barometer.

§ 7. "To fill a tube by boiling, an alcohol-lamp is needed, although it can be done over a charcoal fire. The lamp being filled and put in order, begin to fill

* Williamson on the Barometer, p. 140.

the tube by pouring in through the funnel as much warm mercury as will occupy about five inches; then, holding the tube with both hands above the mercury, heat it gently, and let it boil from the surface of the mercury downward to the end of the tube, and then back again, chasing all of the bubbles of air upward. A little practice will make this easy, the tube being held a little inclined from the horizontal, and constantly and rapidly revolved, always in the same direction, so that every portion of the metal may be heated gradually and uniformly. After this has been done, let the tube cool sufficiently to admit of its being held by the gloved hand, and then pour in enough warm mercury to occupy several inches more of the tube, which may now be held with both hands, one above and the other below the heated portion. After boiling this thoroughly free from air, repeat the same operation with more mercury added, until the tube is filled to the end. With care and practice the mercury may be boiled entirely free from air

up to within an inch or less of the end of the tube. A tube filled in this way may have, in every respect, as perfect a vacuum as one prepared by a professional instrument-maker." *

§ 8. In extended barometric operations in the field, a supply of extra tubes is carried, to be used in case a tube is broken. These tubes should be drawn out so as to be a little longer than they are required to be when fitted into the barometer. The open end should be cut off to such a length that it shall always be immersed and yet not interfere with the rise of the lower part of the cistern. When the instrument is finally put together, the cork in the upper end of the brass case should be adjusted so as to hold the closed end of the tube firmly.*

§ 9. *Cleaning the Barometer.*—It frequently happens that the mercury in the cistern becomes so dirty that the ivory point, or its reflection in the mercury, can no longer be seen; this often occurs

* Williamson, p. 138.

even though the barometer be in good condition in every other respect. The instrument can be taken apart and cleaned with safety and without changing in the slightest degree the zero of the instrument.* Everything used in the operation must be clean and dry. Avoid blowing upon any of the parts, as the moisture from the breath is injurious.

"Screw up the adjusting screw at the bottom until the mercury entirely fills the tube, carefully invert, place the instrument firmly in an upright position, unscrew and take off the brass casing which encloses the wooden and leather parts of the cistern. Remove the screws and lift off the upper wooden piece to which the bag is attached; the mercury will then be exposed. By then inclining the instrument a little, a portion of the mercury in the cistern may be poured out into a clean vessel at hand to receive it, when the end of the tube will be exposed. This is to be closed by the gloved hand, when the instrument can be

* Williamson, p. 136.

inverted, the cistern emptied, and the tube brought again to the upright position. Great care must be taken not to permit any mercury to pass out of the tube. The long screws which fasten the glass portion of the cistern to the other parts can then be taken off, the various parts wiped with a clean cloth or handkerchief and restored to their former position."

"If the old mercury is merely dusty, or dimmed by the oxide, the cleaning may be effected by straining it through chamois leather, or through a funnel with a capillary hole at the end, of a size to admit of the passage of but a small thread of the metal. Such a funnel is conveniently made of letter-paper. The dust will adhere to the skin or paper and the filtered mercury will present a clean and bright appearance. If chemically impure, it should be rejected, and fresh, clean mercury used. With such clean mercury the cistern should be filled as nearly full as possible, the wooden portions put together and securely fastened

by the screws and clamps, the brass casing screwed on, and the screw at its end screwed up. The instrument can then be inverted, hung up, and readjusted. The tube and its contents having been undisturbed, the instrument should read the same as before." *

With the instrument before the operator, these instructions are easily understood. If a little mercury has been lost during the operation, and there is none at hand to replace it, no serious harm has been done; but if much is lost, the open end of the tube may become exposed in inverting the instrument, in which case air may enter. In this case, as in using and caring for any instrument, a little care and a thoughtful inspection of the method of construction is worth more than any written description.

§ 10. *Transporting the Barometer.*—"In transporting a barometer, even across a room, it should be screwed up and carried with its cistern uppermost. For traveling, it should be provided with

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* Williamson, p. 137.

a wooden and leather case. In steamboats or railroads it should be hung up by a hook in the stateroom or car. In wheeled vehicles it should be carried by hand, supported by a strap over the shoulder, or held upright between the legs; but it should not be allowed to rest on the floor of the carriage, for a sudden jolt might break the tube. If carried on horseback it should be strapped over the shoulder of the rider, where it is not likely to be injured, unless the animal is subject to a sudden change of gait. When about to be used it should be taken from its case, while screwed up, gently inverted and hung up, when it can be unscrewed. While it has its cistern uppermost the tube is full, is one solid mass of metal and glass, and not easily injured; but when hung up, a sudden jolt might send a bubble of air into the vacuum at the upper end of the tube, and the instrument would be useless until repaired." *

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* Williamson, p. 134.

§ 11. *Reading the Barometer.*—Read the attached thermometer first; it is more sensitive than the barometer, and the heat of the body affects it, while the barometer is not affected. The thermometer should be read as closely as possible, for a difference of 0.1° F. is equivalent to about 3 feet in altitude. Parallax should be carefully avoided in reading the thermometer.

Then bring, by means of the adjusting screw at the lower end of the instrument, the ivory point just touching the mercury in the cistern. If there is a line of light visible between the point and mercury, the instrument is set too low; if neither a line of light nor a depression can be seen, the adjustment has been correctly made. When the mercury is bright, a shadow of the point can be seen, and if the shadow and the point itself form a continuous unbroken line, there can be no line of light. It is usually best to lower the screw till a distinct line of light can be seen, and then gradually raise it until the light disappears. Before mak-

ing the final adjustment, tap the barometer a little just above the cistern, to destroy the adhesion of the metal to the glass. Complete the contact of the mercury and the ivory point, at the same time being certain that the barometer hangs freely, *i.e.*, vertically.

Next tap the barometer gently in the neighborhood of the top of the mercury column to destroy the adhesion of the mercury; this is very important, since raising or lowering the mercury in the previous operation materially affects the form of the upper surface. Then take hold lightly of the brass casing of the barometer, not near the attached thermometer, so as not to unnecessarily heat either the case or the thermometer, and by means of the mill-head screw near the middle of the tube, bring the front and back edge of the vernier into the same horizontal plane with the top of the mercury in the tube, just touching it and no more, and then remove the hand. Move the eye about, and if, in any position, a line of light can be seen between

the mercury and the vernier, the latter must be moved down a little; if there is no line of light, but a large space is obscured, the vernier must be moved up a little. As the top of the column is more or less convex, when the adjustment is correctly made a small place is obscured in the center, when the light is seen on either side.

Finally having adjusted the instrument as above, it may be read at leisure. On the best barometers the scale is usually divided to inches, tenths, and half-tenths; the vernier reads to one twenty-fifth of half-tenths ($\frac{1}{25} \times 0.05$) or two-thousandths (0.002).

## ART. 2. THE THEORY.

### A. COMMON OR STATICAL FORMULA.

§ 12. *Fundamental Relations.*—Suppose A and B, Fig. 2, to represent two stations, and that it is required to determine the vertical distance between them. A and B are not necessarily in the same vertical line. Let C represent any point in A B, and D a point a small distance below C.

Suppose the pressure per square inch at D to be represented by P, and the difference in pressure between C and D by $d$P. Let $a$=the weight of a cubic inch of air

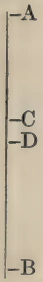


FIG. 2.

under the conditions of pressure, temperature, etc., existing between C and D; X= the elevation of A above B, in feet.

It is clear that the increase in pressure from C to D is equal to weight of a column of air between C and D whose cross section is 1 sq. in.; or,

$$adx = d\mathrm{P} \qquad (1)$$

§ 13. If $a_0$=the weight of a cubic inch of dry air at the sea-level in latitude 45° at the freezing point when the barome-

ter stands at 29.92 inches, and $P_0$=the pressure under which $a_0$ is determined, then by Boyle and Marriott's law

$$a:a_0::P:P_0;$$

or
$$a=a_0\frac{P}{P_0} \qquad (2)$$

If H and $H_0$ represent the heights of the barometer assuming the temperatures to be the same, corresponding to the pressures P and $P_0$ and $m$=the weight of a cubic inch of mercury, then

$$\frac{P}{P_0}=\frac{Hm}{H_0m}=\frac{Hm}{29.92m} \qquad (3)$$

and
$$a=a_0\frac{P}{P_0}=\frac{a_0Hm}{29.92m} \qquad (4)$$

The weight of a cubic inch of air for any other temperature $t$ is

$$a=\frac{a_0}{m}\frac{Hm}{29.92}\frac{1}{(1+ct)} \qquad (5)$$

in which $c$ is the coefficient of expansion of air. For any latitude $\phi$, (5) becomes

$$a=\frac{a_0Hm}{29.92m}\frac{1}{(1+ct)}\frac{1}{(1+0.0026\cos.2\phi)}$$

very nearly. (6)

§ 14. Substituting the value of $a$ from (6) in (1), and $m d\mathrm{H}$ for $d\mathrm{P}$, we have

$$dx = 29.92\frac{m}{a_0}(1+ct)(1+0.00260\cos.2\phi)\frac{d\mathrm{H}}{\mathrm{H}}$$

which integrated between the limits H′ and $\mathrm{H}_1$, the heights of the barometer at A and B respectively, gives

$$\mathrm{X} = 29.92\frac{m}{a_0}(1+ct)\ (1+.00260\cos.2\phi)$$
$$\mathrm{N}\log.\frac{\mathrm{H}_1}{\mathrm{H}'} \qquad (7)$$

Assuming the mean of the temperature of the air at A and B to be the mean temperature of the air between A and B, we may put

$$t = \frac{\mathrm{T}'+\mathrm{T}_1}{2}, \qquad \mathrm{T}' \text{ and } \mathrm{T}_1$$

being the temperatures of the air at A and B. Making this substitution and passing to common logarithms,

$$\mathrm{X\,ft.} = 5.74\frac{m}{a_0}\log.\frac{\mathrm{H}_1}{\mathrm{H}'}\left(1+c\frac{\mathrm{T}'+\mathrm{T}_1}{2}\right)$$
$$(1+0.0026\cos.2\phi) \quad (8)$$

This formula includes the principal relations involved in determining difference

of height with the barometer. The final formula to be used in practice has been given differently by different investigators, according to the values chosen for the constants, to the individual preference for one form over another, and to the degree of refinement desired. A few of these special formulas will now be considered briefly.

§ 15. *The Constants.*—The value of the term $5.74 \ldots \frac{m}{a_0}$, generally known as the barometric coefficient, will depend upon whose values of the densities of air and mercury are used. Boit and Arago found* $\frac{m}{a_0} = 10467$, which makes the barometer coefficient 60,096.3 ft. (18,317 meters). Regnault's values* which are the most recent and probably the most accurate, give 60,384 feet (18,404.8 meters). Raymond (1803) found* the value of the barometric coefficient by determining the value it should have to make the results by the formula agree with those

* Smith, Miscel. Col., Vol. I., Pt. IV., p. 9.

furnished by trigonometrical leveling. The value obtained in this way is 60,158.6 ft. (18,336 meters); but even under the most favorable circumstances, the observations, eight in all,* were too few to determine such a coefficient with sufficient accuracy. For reasons which will appear in this and Chapter IV., it is highly probable that Raymond's coefficient is the least accurate, although it has been more frequently used than either of the others.

The term $\left(1+c\frac{T'+T_1}{2}\right)$ is known as the temperature term. $c$ is the coefficient of expansion of air, and is equal to 0.00375 per 1° C.; it is usually approximated at 0.004. If this value be substituted, the temperature term becomes $\left(1+\frac{2(T+T)_1}{1000}\right)$ for centigrade degrees; if $T_1$ and $T'$ are given in Fahr. degrees, it is easily seen that the temperature term becomes $1+\frac{T_1+T'-64}{900}$.

* U. S. C. & G. Report, 188?, p. 235.

The term $(1+0.00260 \cos. 2\phi)$ is known as the latitude term. A few formulas still contain an older and less accurate coefficient of $\cos. 2\phi$ than the above.*

§ 16. *Laplace's Formula.*—Laplace was the first to give a rational formula for determining heights by the barometer, and his formula has served as a basis for several others. It differs from equation (8) as above in having a correction for the variation of gravity in the vertical. If the altitude of the lower station about the sea-level be represented by $z$, then the weight $a$ at the sea-level as given by (6) must be multiplied by $\dfrac{R^2}{(R+z+x)^2}$ to make it true for any altitude. This leads to an equation which can be integrated approximately by developing the above factor into a series. All the approximations leading to Laplace's form are not easy to discover, and the importance of the matter does not warrant a long search. Laplace's complete formula is †

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* U. S. C. & G. Report, 1881, p. 227, eq. (6).

† Williamson, p. 164; Guyot's Col., etc.

$$X = 60158.6 \log. \frac{H_1}{H'} \left\{ \begin{array}{l} \left(1+\frac{T'+T_1-64}{900}\right) \\ (1+0.0026 \cos. 2\phi) \\ \left(1+\frac{X+52252}{20886860}+\frac{z}{10443430}\right) \end{array} \right. \quad (9)$$

In which X is in feet and the temperatures in Fahrenheit degrees; X in the last term is the value of the preceding part of the formula.

Since the entire correction for the variation of gravity is always quite small, and since at best the barometer can be read only to thousandths of an inch, which corresponds to about 10 ft. of altitude, the latitude term and also the term for variation of gravity with the altitude may be neglected without materially affecting the accuracy of the results. Farther on it will be shown that the appearance of extreme accuracy by retaining these terms can be regarded only as a mathematical illusion, inapplicable to any real state of practice.

§ 17. *Babinet's Formulas.*—The following formula* by Babinet has no term for

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*** Smithsonian Miscel. Col., Vol. I, Part IV. (Guyot's Collection), p. 68.**

the variation of gravity. It is sometimes claimed* that the barometric coefficient was adjusted to meet this correction; but from the nature of the case this can not be true, except for some assumed mean. However, notice that the coefficient is larger than any previously given, see §15. The formula is, for X in feet, and Fahrenheit degrees,

$$X\,\text{ft.} = 60334 \log. \frac{H_1}{H'} \left\{ 1 + \frac{(T' + T_1 - 64)}{900} \right\} \tag{10}$$

If $H'$ and $H_1$ do not greatly differ it can readily be found that

$$N \log. \frac{H_1}{H'} = 2\frac{H_1 - H'}{H_1 + H'}$$

Making this substitution in (10) gives Babinet's approximate formula

$$X\,\text{ft.} = 52{,}400\frac{H_1 - H'}{H_1 + H'} \left\{ 1 + \frac{(T' + T_1 - 64)}{900} \right\} \tag{11}$$

"The error involved in the above formula is inappreciable for elevations less than 3,000 feet."†

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* Ibid., p. 9. † Ibid., p. 68.

The following* is essentially the same as Babinet's approximate formula except the form of the temperature term

$$X = 54500\frac{H_1 - H'}{H_1 + H'} + \frac{T_1 + T' - 110}{900}$$
$$\pm \frac{X}{200} \pm 10 \text{ ft.} \quad (12)$$

The last two terms show the degree of reliance to be placed upon the result.

§ 18. *Correction for Temperature of Barometer.*—In all that has preceded, it has been assumed that the two barometers were at the same temperature, which assumption will rarely or never be true. Therefore the heights of the barometer, before being inserted in the preceding formulas, must be reduced to the corresponding heights which they would have at a common temperature, or a term must be included in the formulas themselves to correct for the difference in temperature of the barometers. Both methods are employed.

The expansion of mercury for 1° F. is

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* U. S. C. & G. Report, 1876, pp. 352–3; see also Lee's Table, p. 151 (3d ed.).

0.000,1000, and that of brass, of which the scales are generally made, is 0.0000,-104; the difference, the relative expansion of mercury, is 0.0000896. For the centigrade scale this difference is 0.00016141. Hence if $h'$ represents the height of the barometer at the upper station, reduced to the temperature of the lower, and $t'$, $t_1$ the temperature of the barometers at the upper and lower stations respectively, we have

$$h' = \mathrm{H}'[1 - d(t' - t_1)], \qquad (13)$$

in which $d$ stands for one of the above differences, according to the kind of thermometer used.

§ 19. Instead of reducing one barometer to the temperature of the other, both may be reduced to any other temperature assumed as a standard; the freezing point of water is generally chosen. Equation (13) is still applicable, provided $t_1$ be considered as representing the temperature of melting ice (32° F. or 0° C.), and $t'$ the reading of the attached thermometer. The formula for reduction must now be applied to both read-

ings of the barometer. Numerous tables have been computed for facilitating this reduction — see Guyot's Collection, 3d Edition, Group C, pp. 61–127; Group D, pp, 30, 46, 53, etc.; Lee's Tables, 3d Ed., pp. 152–9; Williamson on the Use of the Barometer, pp. 1–64 of the Appendix, etc.

§ 20. The correction for the difference in temperature of the barometers may be made by inserting a term in the general formula. Thus, in (8), it is only necessary to multiply H′ by $[1-d(t'-t_1)]$ to reduce H′ to the corresponding height at the temperature of the lower barometer. Making this correction, using Babinet's barometric coefficient (§ 17) and approximating $d$ (.0000896) at .0001, we get *Bailey's Formula.**

$$\mathrm{X\,ft.} = 60346 \log.\frac{\mathrm{H}_1}{\mathrm{H}'} \times \frac{1}{1-.0001\,(t'-t_1)} \left\{ \begin{array}{l} 1+\dfrac{\mathrm{T}'+\mathrm{T}_1-64}{900} \\ 1+0.0026 \cos.\ 2\phi \end{array} \right. \tag{14}$$

§ 21. *Correction for Humidity.*—In deducing the preceding formula, it was assumed that the atmosphere was com-

* Guyot's Collection, D, p. 69.

posed exclusively of dry air; really it is a mixture of air (oxygen and nitrogen), carbonic acid, and watery vapor. The carbonic acid is very small and nearly constant, and hence it need not be considered here; but the watery vapor is both large and variable. If dry air and aqueous vapor had even nearly the same density under the same conditions, the presence of the latter would not affect the problem; but watery vapor is only five-eighths as dense as dry air, and the weight, *a*, of a unit of volume of the atmosphere will depend upon the relative amount of vapor which it contains. Accurate hypsometry accordingly demands that some account shall be taken of the aqueous contents of the atmosphere, and a humidity term has been included in many barometric formulas.

The introduction of a humidity term in the barometric formula requires that the hygrometric state of the air column shall be known. Accordingly an observation with the hygrometer is made at each station. For this purpose the wet bulb

hygrometer or psychrometer is generally preferred, because of its greater accuracy and convenience; knowing the readings of the wet and dry bulb thermometers, the barometric pressure due to the aqueous vapor in the air may be determined from tables,* which are the results of experiments. The observed heights of the barometer may be corrected for the pressure of the aqueous vapor before substituting them in the formula; or the observed heights may be used uncorrected, and the resulting altitude be multiplied by a factor to correct for the humidity. The latter method seems to be generally preferred.

"In a very general sense, in temperate climates near the sea-level the amount of vapor in the atmosphere is from 0.2 to 0.4 of an inch, or about one-hundredth of the whole:

§ 22. Bessel was the first to propose the introduction of a correction for the effect of moisture. Plantamour's for-

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[1] * Williamson on the Barometer, Table C, of the Appen.; Guyot's Col., Group B, pp. 46–72, pp. 102–6.

mula,* which differs from the one proposed by Bessel only in the form and the value of the constants adopted, and Rhulmann's formula† are frequently used and are representative of this class. Williamson ‡ has translated Plantamour's formula into Laplace's form [9].

§ 23. It is doubtful whether any considerable increase of accuracy is obtained by including a separate correction for the aqueous vapor.§ The laws of the distribution and transmission of moisture through the atmosphere are too little known, and its amount, especially in mountain regions, is too variable and depends too much upon local winds and local condensation, to allow a reasonable hope of obtaining the mean humidity of the layer of air between the two stations by means of hygrometrical observations taken at each of them. The difficulty lies in getting the mean humidity of the vertical column between the two stations. The observations for humidity are made

* Guyot's Col., D, p. 72.
† 3 U. S. C. S. Report, 1876, p. 350.
‡ On the Barometer, pp. 100–6 of the Appendix.
§ Guyot's Collection, D, p. 33.

in the stratum of air next to the surface of the earth, which probably contains the greatest amount of moisture, and which is therefore least representative of the vertical column between the two stations. At any rate, the gain, if there is any, is not sufficient to compensate for the extra trouble in making the observations and the undesirable complication of the formula.

The question of the desirability of applying a correction for the hygrometric state of the atmosphere is so intimately connected with the phase discussed in division B of this article, which immediately follows, that nothing farther need be said here.

§ 24. *Conclusion.*—The preceding do not comprise all the formulas which have been proposed for barometric leveling, but include the more common ones, and illustrate all the principles involved, except those discussed in division B. Some of the omitted formulas are approximate, some have empirically determined pressure coefficients, etc. Owing to the limi-

tations discussed in the next division, "it matters comparatively little which of the generally recognized barometric formulas is used."

B. Dynamical Formulas.

§ 25. *Defects of Statical Formulas.*—All the formulas referred to above are dependent upon the assumption that the air is in a state of statical equilibrium. If a condition of statical equilibrium were possible, we might suppose that the whole atmosphere was arranged in a system of horizontal layers, each of which would be denser than the one above it and rarer than the one below, each being uniform throughout in temperature and humidity. The temperature and humidity might vary from stratum to stratum uniformly, or according to some more complicated law.

The fundamental assumption in deducing the preceding class of formulas is: 1, that a difference of pressure is due only to a difference of elevation; 2, that the temperature of the air varies uniformly

from one station to the other; and 3, that the temperature of the air between the two stations is the same as that of the vertical column between the horizontal planes of the two points. The introduction of a correction for humidity involves essentially the same assumptions as the temperature term.

§ 26. The air is never in a state of statical equilibrium, but is perpetually undergoing local changes of pressure, temperature, and humidity. "For example, the sun, which is the ultimate source of all disturbances, shines only by day. While it shines a certain amount of heat is imparted to the whole atmosphere, but a much higher temperature is given to the ground, and is communicated to the contiguous layer of air. At night the atmosphere loses heat by radiation to space, but the ground loses it still more rapidly and imparts its low temperature to the lowest stratum of air. The lower strata, therefore, have exceptional warmth by day and exceptional coolness by night. If the air is moist it intercepts a greater

quantity of solar heat than if it is dry, so that a less quantity reaches the ground, while at night atmospheric moisture checks radiation from the ground. The power of the earth's surface to receive or store or part with heat varies with its character. Naked rocks and cultivated fields, bare earth and grass, forest and snow are affected very differently by the heat rays of the sun, and exert equally diverse influences on the adjacent air, so that one tract of land is often in a condition to heat the air while an adjacent tract is cooling it. Then, too, the sun's heat is unequally distributed through the year; outside the tropics there is a progressive accumulation of heat through summer and a progressive loss through winter. The ocean undergoes less change of temperature than the land, and its rate of change is slower, so that there is frequent, and indeed almost continuous, contrast of condition between it and the contiguous land. As a result of all these influences, together with others that might be enumerated, the equilibrium of

the air is constantly overthrown, and the winds, which tend to readjust it, are set in motion.

The temperature of the air is continually modified by external influences; the static order of densities is broken and currents are set in motion; and the circulation and the inequalities of temperature conspire to produce inequalities of moisture. Every element of equilibrium is thus set aside, and the air is rendered heterogeneous in density, temperature, and composition." *

§ 27. A consideration of these facts will show the inaccuracy and insufficiency of hypsometric formulas founded upon an assumed state of static equilibrium. Some of the defects of statical formulas will be considered in detail before discussing formulas which seek to overcome these difficulties.

§ 28. *Gradient.*—Let A, B, and C designate three barometer stations. Let A′, B′, C′ designate points vertically above each at which the pressure is the same or

* G. K. Gilbert, in U. S. Geological Report for 1880–1.

common. The plane passing through A′ B′ C′ is then a surface of equal pressure. If the air were in a state of equilibrium, it would be a level plane, but in fact it will be inclined in some direction. This inclination is called the *barometric gradient*.

Instead of considering only three points, "we can in imagination project through the air a surface containing all points which have the same pressure. If the atmosphere were at rest, this surface would be a horizontal plane, but under the actual conditions it is never a plane and is ever undulating." For small areas under ordinary conditions, this surface would probably not differ much from a plane.

Conceive another surface passed through all points at which the pressure differs from the preceding one by any constant quantity. With atmospheric equilibrium all such surfaces would be both level and parallel, but in the actual case none are level and no two are precisely parallel. When widely separated surfaces are com-

pared, the variations from parallelism are often so great that their inclinations above the same locality have opposite directions. The atmospheric gradient at the surface of the ground may therefore differ greatly in amount and direction from the simultaneous gradient at a considerable altitude above the same spot.

§ 29. The necessity of considering the barometer gradient is apparent when it is remembered that the air is continually in a state of motion, as is shown in the variation of the height of the barometer. For example, if A and B are two stations, and the atmosphere at rest, then the surface of equal pressure, BC, is a horizontal plane; and AC is the difference of elevation which would be obtained by applying any one of the preceding barometric formulas. If the air is not in static equilibrium, the pressure at A will be greater or less than before, and the surface of equal pressure may lie above or below BC; if the pressure at A is greater then the average, the surface of equal pressure is above C, say at E, and

AE is the corresponding difference of elevation obtained by applying a statical formula.

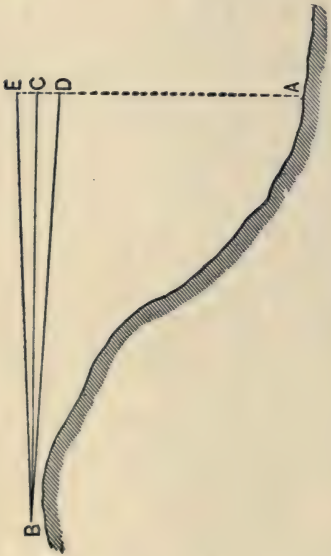


FIG. 3.

The problem is farther complicated by the fact that the air above B also is in a state of oscillation. If the variations in pressure at the two stations were simul-

taneous and alike in amount, no error would be produced by the barometer gradient; but these conditions are seldom or never realized.

§ 30. The variations in atmospheric pressure, and the consequent variations of gradient, are so complicated that it is impossible to trace the relation between cause and effect; but there are two variations that are pretty well understood. One has a daily period, and is caused by the variation in the heating effect of the sun between day and night; the second has a yearly period, and is caused by the variations of the sun's heat at different times of the year.

§ 31. *Diurnal Gradient.*—It is a fact familiar to meteorologists that the pressure of the air everywhere undergoes a daily oscillation. The gradient introduced by this daily change is called diurnal gradient. The pressure has two maxima and two minima which are easily distinguishable. Near the sea-level the barometer attains its maximum about 9 or 10 A. M. In the afternoon there is a

minimum about 3 to 5 P. M. It then rises until 10 to midnight, when it falls again until about 4 A. M., and again rises to attain its forenoon maximum; the day fluctuations are the larger.

The daily oscillation is subject to variations in character and magnitude. The oscillation is greatest at the equator and diminishes toward the poles, but is not the same for all places of the same latitude. Within the United States it varies between 40 and 120 thousandths of an inch. Changes of altitude often cause a marked variation in the amount and character of the diurnal oscillation. The difference which pertains to latitude does not materially affect the ordinary hypsometric problem, but the difference depending on the altitude has a very important effect.

§ 32. *Annual Gradient.*—The annual progress of the sun from tropic to tropic throws a preponderance of heat first on one side of the equator and then on the other, which produces an annual cycle of changes in the pressure and gives rise to what has been called the annual gradi-

ents. The amount of this variation is quite small near the equator, but increases rapidly toward the poles; "at the equator it rarely exceeds ¼ of an inch per year, while in the polar regions it is often as much as 2 or 3 inches in a few days." *

§ 33. *Non-periodic Gradients.*—In addition to the diurnal and annual variations in the pressure, there are others due to the same general cause, the heat of the sun, but so modified by the local conditions—topography, the humidity, winds, storms, etc.—as to make it impossible to discover the law of their action. These non-periodic variations are much greater in amount and more rapid in their actions than either of the others.

§ 34. *Temperature Gradient.*—"It has just been explained that the variations of pressure are due primarily to inequalities of temperature; it will now be shown that, if differences of elevations are determined by the formulas commonly used,

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*Williamson, p. 68.

the temperature is directly responsible for other and generally more serious errors." They arise from the difficulty


FIG. 4.

of determining the temperature of the vertical column of air between the two stations.

Let A and B (Fig. 4) be two stations the difference of elevation of which is to be obtained from observations of the barometer and thermometer made at each. Let it be assumed that the pressure observed at B is the same as that at C vertically over A and on a level with B. To use the statical or common formulas, the temperature of the column AC must be known; in applying these formulas it is assumed that the mean temperature of this column is equal to the mean of the temperatures observed at A and B.

"How admissible this assumption is will appear at once when the manner in which the air acquires and loses heat is recalled. The body of the atmosphere is heated directly by the sun and gives off its heat by radiation into space. The surface of the earth is heated and cooled in the same manner, but many times more rapidly, so that by day it is always much warmer than the body of the air, and by night it is much cooler. A layer of air next to the earth receives its warmth

from the earth, and is thereby caused to differ widely in temperature from the remainder of the atmosphere. Not only is the greater part of the column inaccessible to us, but that portion to which our observations are restricted is the portion least representative of all." *

By measuring the difference of elevation of two points with the spirit level reversing the barometer formula, and computing the temperature of the air column, it has been found that in middle latitudes the average daily range of the temperature of the body of the air is about 4°, and that of the superficial layer, is from 10° to 20° near the seashore, and from 20° to 35° in the interior of continents. There is a stratum of air near the surface (Fig. 3) which oscillates daily through this wide range, while the temperature of the upper and lower portion of the column AC is relatively constant. Therefore the mean of the observed temperatures absolutely

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* Gilbert in U. S. G. S. Report.

fails to give the mean temperature of the column AC as required by the formula.

§ 35. Nor does the trouble end here. "Whenever the ground layer is cooler than the air above, it is of course heavier, and, like any other heavy fluid, it flows down hill and accumulates in valleys, forming lakes of cold air. The nightly layer of abnormally cool air is therefore thinner on eminences than in valleys, and the contrast increases as the night advances. When the conditions are reversed so that the ground layer is warmer than the air above it, it has a tendency to rise, but accomplishes the change in an irregular manner, breaking through the immediately superior layer here and there and rising in streams which spread out in sheets wherever the conditions of equilibrium are reached." Observers in balloons, as they ascend or descend, rarely find an orderly succession of temperatures. If, therefore, we could in some way determine the temperature of some point in the upper portion of the

column AC, we should still be unable to deduce the mean temperature of the column with a high degree of accuracy."

§ 36. *Humidity.*—The errors in barometric leveling due to the moisture in the air are essentially the same in kind as those due to the uncertainty of the temperature. The observations are made in the stratum next to the earth in which the amount of moisture is the greatest and the most variable. A change of station of a few feet, or a slight variation in the direction or force of the wind, will often cause a very important difference in the amount of watery vapor. The variations in moisture of the different portions of the atmosphere are greater than the variations in temperature. The diffusion of aqueous vapor is so slow that its effect may be neglected, and the distribution considered as taking place only by the circulation of the air. For this reason, then, there will be the same heterogeneity of moisture as of temperature, which has already been discussed.

"The variations in the hygrometric

state are still further increased by the laws of condensation. At the surface of the earth there is an almost continuous passage of moisture from ground to air, only a part of the total exhalation being returned as dew. The daily circulation incited by the heat of the sun carries the moistened air upward and eventually the water is returned to the earth in the form of rain or snow, but the condensation and succeeding precipitation are exceedingly irregular. Whenever, therefore, a current of air moves upward and its temperature is lowered by rarefaction, a point may be reached where the accompanying vapor can no longer exist as such, and is condensed to cloud or even to rain or snow. On the other hand, whenever a current of air moves downward, its capacity for moisture is increased, and it acquires a quasi-absorbent power, so as to take up water from whatever moist surface it touches."

The irregularities of humidity are greater proportionally than the associated irregularities of temperature, but the

error in the difference of altitude due to humidity is less than that due to temperature, because humidity is a much smaller factor of hypsometric problems.

§ 37. *Ferrel's Formula.*—Ferrel* has deduced from a consideration of dynamical principles, a barometric formula which distinctly recognizes the defects, as discussed above, of formulas founded upon a statical condition of the atmosphere, and which indicates a method of remedying them. Although the formula is very carefully and ingeniously worked out, yet it is probably of little use for ordinary hypsometrical work, since it requires observations to be made for a long time over a considerable area, to get the data by which to compute corrections for gradient, temperature, and humidity.

Without the data for making these reductions, this formula is essentially the same and has essentially the same defects as the formulas depending upon a statical condition of the atmosphere.†

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* U. S. C. S. Report, 1881, pp. 225–68.
† Wm. Ferrel in U. S. C. & G. S. Report, 1881, p. 243.

§ 38. *Gilbert's Formula.*—Gilbert* has developed a method for determining heights with the barometer which does not require observations of the temperature and humidity of the air. His method requires simultaneous observations of the barometer at three stations, the vertical distance between two of which is known; from the known difference between two of the stations and the observations at each, the actual density of the air can be found; then the true density can be used to compute the difference of elevation between either of these stations and the third.

The method is most accurate when the three stations are in the same vertical and when the one whose elevation is desired lies between the two, the difference of whose elevations is known; the method is applicable, but is less accurate, when the stations are not in the same vertical, or when the one whose height is sought lies either above or below the other two.

* In U. S. Geol. S. Report, 1880–1.

"One of the distinctive characteristics of this method is that it observes density directly, whereas other methods observe temperature and moisture only and deduce density. The only reason which has ever existed for measuring the temperature of the air and the moisture in it has been to ascertain its density. A second distinctive feature is that this method employs in its determination of density a column of air comparable in height with the one to be measured and fairly representative of it, while other methods base their diagnosis of the column to be measured on density determinations made close to the ground, where, as a rule, the conditions are not representative." It will be shown presently that this method also has serious defects.

§ 39. The formula for this method is deduced as follows:

Let L, N, and U represent the altitudes of the lower, new, and upper stations respectively; let $l$, $n$, and $u$ represent the synchronous barometric readings at these

same stations corrected for temperature of the instrument and instrumental errors. Let $B =$ the vertical base line, or the known difference of altitude of the upper and lower stations, $B = U - L$. Let $A =$ the required difference of altitude $N - L$, and let $a =$ an approximate value of A.

Since $B = U - L$,

and $A = N - L$, $B - A = U - N$.

For convenience refer all vertical distances to the lower station as an origin.

If for the present we neglect the decrease in temperature and moisture with an increase of the altidute, and assume that the accidental or temporary variations of density due to temperature and humidity are the same in both columns, the following proportion may be made:

$$\left.\begin{array}{c}\text{The approximate}\\ \text{height of the}\\ \text{base line}\end{array}\right\} : \left\{\begin{array}{c}\text{the true height}\\ \text{of the base}\\ \text{line, B,}\end{array}\right\}$$

$$:: \left\{\begin{array}{c}\text{the approximate}\\ \text{height of the}\\ \text{new station}\end{array}\right\} : \left\{\begin{array}{c}\text{the true h'ght}\\ \text{of the new}\\ \text{station, A.}\end{array}\right.$$

The approximate height (length) of the base line, as deduced from the readings of the barometer at the two stations is, C (log. $l-$log. $u$) in which C is the barometer constant. In the same way the approximate height of the new station above the lower is C (log. $l-$log. $n$). Substituting these values, the above proportion becomes,

$$C\,(\log. l-\log. u) : B :: C\,(\log. l-\log. n) : A.$$

or $a=B\,\dfrac{\log. l-\log. n}{\log. l-\log. u}$ (15), in which $a$ is written instead of A, owing to the neglect of the variation of temperature and humidity with the altitude.

§ 40. The preceding equation would give the correct result if the atmospheric column were uniform in temperature, and if its aqueous vapor were uniformly distributed; but since this is never the case there must be added a term which shall take account of the variation of temperature and moisture with the altitude.

It is well known that in a general way the temperature and moisture decrease with the altitude; but the exact law of this variation has not yet been discovered. Therefore, before the correction to be added to equation (15) can be determined, it will be necessary to assume some law for this variation.

"If the air were of uniform density, and the element of temperature were introduced alone, the high temperatures at low altitudes would cause a dilation there, and the low temperatures at high altitudes would cause a contraction, and the resulting distribution of densities would be characterized by an increase from below upward. If the air were of uniform density and the element of vapor distribution were introduced alone, the greater per cent. of aqueous vapor (which is a rarer gas than dry air) in the lower strata, would cause them to be relatively rare, and the resulting distribution of densities would be characterized by an increase from below upwards."* Con-

* U. S. Geological Survey Report, 1881, p. 441.

sequently Gilbert* assumes that the increase in density due to temperature and humidity varies directly as the altitude. This may not be the true law, and whenever a better assumption becomes possible it should be introduced instead of the one here used.

In Art. 3 tests of the complete formula will be given, which will afford an idea of the admissibility of this assumption.

§ 41. To embody this correction in the formula, let D represent the vertical distance in which the increase of density due to temperature and humidity is equal to the density at the ground; then the increase of density for each unit of vertical space is expressed by $\frac{1}{D}$. The mean density of the volume of air between the upper and lower station, in so far as it depends upon temperature and humidity, occurs at its middle point, which is a distance $\frac{B}{2}$ above the lower station. Likewise the mean density of the column be-

* U. S. Geological Survey Report, 1881, p. 441.

tween the lower and the new station is at a point $\frac{A}{2}$ above the lower. The vertical distance between these points is $\frac{B}{2}-\frac{A}{2}$. The decrease of density from the middle point of the column A to the middle point of the column B is $\frac{B-A}{2}.\frac{1}{D}$; that is, the mean density of the column A is $\frac{B-A}{2D}$ greater than that of the column B.

In deducing the first term (15) of the proposed formula, it was assumed that the density as far as it depended upon temperature and humidity was uniform throughout both columns; but we have just shown that the element of the density varies directly as the altitude; consequently a term must be added to (15) to correct for the variation. It has just been shown that the mean density of A is $\frac{B-A}{2D}$ greater than that of B. The mean density of B is the unit or standard density; consequently, to express the density

of A in terms of B, the density of A, as assumed in (15), must be diminished by $\frac{B-A}{2D}A$.

Finally, the neglect of the variation of density with temperature and humidity assume too great a density for column A, and since heights are inversely proportional to densities, that which makes the density too great makes the height too small; therefore the height of A as deduced from (15) must be increased by the quantity $\frac{B-A}{2D}A$.

Adding this term to (15) gives for the height A of the new station,

$$A=B\frac{\log. l-\log. n}{\log. l-\log. u}+A\frac{(B-A)}{2D} \qquad (16)$$

Since the last term is always small, *a* can be substituted for A, thereby making the formula more convenient to compute.

§ 42. If the position of the new station be referred to the upper station instead of the lower, the above formula will remain unchanged, except *l* and *n*, and *l* and *u* will change places.

The formula is applicable, even though the new station is not intermediate in height between the other two. If the new station is above both the others, the quantity (B – A) then becomes minus, and the last term is subtracted; if the new station is below both the other two, the numerators of both fractions will be minus, and the result will be the sum of the two terms.*

§ 43. The quantity D can be found only by experiment; to find it, observe the barometer at all three stations, determine A and B by a spirit-level, and compute D. The experiments should cover a great range of conditions so as to secure a fair mean value. Unfortunately D has not yet been determined from sufficiently varied conditions. The only value known is one determined by Gilbert from observations at only two sets of stations, and one of them was not very satisfactory. In this way it was found that D = 245,000 feet.

The internal evidence of the observa-

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* Gilbert, p. 442.

tions from which this value is derived is such that it is probable "its real value will eventually be found to be somewhat smaller than the one provisionally assigned." *

Happily, the last term is always relatively small, and hence any uncertainty in the value of D will have only a small effect upon the final result. The uncertainty in the value of D is the chief defect in this formula. Introducing this value of D, (16) becomes

$$\text{A(in feet)} = \text{B}\frac{\log. l - \log. n}{\log. l - \log. u} + \frac{\text{A(B}-\text{A)}}{490{,}000} \quad (17)$$

$$\text{A(in meters)} = \text{B}\frac{\log. l - \log. n}{\log. l - \log. u} + \frac{\text{A(B}-\text{A)}}{149{,}349} \quad (18)$$

§ 44. *Reduction Tables.*—The use of all the preceding formulas is very much simplified by tables which facilitate their application. Guyot's Collection (Smithsonian Miscellaneous Collection, Vol. 1) contains tables for the application of all the principal statical formulas. The Ap-

* Gilbert, p. 502.

pendix of Williamson *On the Use of the Barometer* contains a series of tables for La Place's form of Plantamour's formula (§ 22), with Regnault's coefficient. The same tables are also given in Lee's Tables, pp. 148–182. The U. S. Geological Survey Report for 1881 contains the only table necessary in applying Gilbert's formula.

Tables are useful where a great number of observations are to be reduced; but they generally contain an unnecessary number of figures, and hold forth a show of extreme accuracy which the nature of the observations themselves can not justify.

§ 45. In the succeeding article it will be shown that statical formulas are generally applied in such a way as to largely eliminate the defects referred to in this section. It is impossible to completely eliminate the errors due to gradient, temperature, etc., and, consequently, difference of elevation cannot be determined with precision by means of the barometer.

## Art. 3. The Practice.

§ 46. *Common Method.*—Results may be obtained by using only one barometer, which is carried from station to station, one or more observations being made at each station; but results obtained in such a manner would be only rude approximations, owing to errors of gradient. The greater the distance between the two points the greater the error. Distant stations are sometimes connected by intermediate ones.

The errors due to change of gradient are partially eliminated by making simultaneous observations at the two stations. If the phase and the amplitude of the variation were the same at both stations, which probably seldom or never occurs, simultaneous observations would give results independent of this class of errors. Errors due to gradient are still further reduced by making a number of simultaneous observations and using the mean; this eliminates only the variable element and fails to take account of permanent gradient.

It is often recommended that the observations be made at certain hours of the day, at which time it is supposed the diurnal and annual gradients are zero. These times can only be determined from experiment, and will vary with the state of the atmosphere, the season, the locality, the elevation, etc. The U. S. Coast Survey * recommend the following times, subject to the preceding limitations. They were probably deduced for the middle Atlantic coast. The hours refer to the middle of the month, other times are to be determined by interpolation.

| | | | | | |
|---|---|---|---|---|---|
| January.................. | | | | 1 | P.M. |
| February.....| 10 | A.M. | and | 4 | " |
| March........ | 8 | " | " | 6 | " |
| April......... | 7.30 | " | " | 7 | " |
| May.......... | 7 | " | " | 7 | " |
| June......... | 6.30 | " | " | 9.30 | " |
| July.......... | 6.30 | " | " | 9.30 | " |
| August....... | 7 | " | " | 7.30 | " |
| September.... | 8 | " | " | 6 | " |
| October...... | 10 | " | " | 3.30 | " |
| November.... | 10.30 | " | " | 2.30 | " |
| December.....at no time. | | | | | |

* Report, 1876, p. 349.

§ 47. Observations made by the preceding method must be reduced by some of the statical formulas. Notice that the preceding methods do not eliminate the error due to the fact that the mean of the observed temperatures does not represent the mean temperature of the air column.

§ 48. *Williamson's Method of Eliminating Gradient Errors.**—This method is specially adapted to reconnoissances and topographical surveys. A centrally located station, called a base station, is chosen, at which the barometer is read at stated hours each day for several days. In the meantime, itinerary observers make observations at the various points, the elevations of which are to be determined, and take pains to have their observations correspond in time with one of the observations at the base station. In the progress of the itinerary survey, a series of observations, similar to those at the base station, are made as frequently as practicable at semi-permanent camps;

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* **Williamson on the Barometer.**

the object of both series being to ascertain the nature of the diurnal variation of pressure and temperature.

The barometric readings at the base stations, corrected for temperature of the instruments, are plotted upon ruled paper so as to exhibit their curve, and all readings shown by inspection to be influenced by abrupt and violent atmospheric disturbances, such as thunder-storms, are discarded, their places being filled by interpolations. From the corrected observations at the base stations, a correction is deduced, which, being applied to the several barometric readings, reduce them to the daily mean; applying this correction eliminates at least part of the effect of diurnal gradient.

Instead of determining the temperature of the air column from the temperature at the time of observing, the mean temperature of the day is used; this can be quite accurately determined at the base stations, but is only approximately known at the other stations. Notice that the mean of the daily means will not be the

mean temperature of the vertical air column.

The difference of altitude can then be computed from the reduced barometric readings and the mean daily temperature, by using any of the statical formulas; Williamson himself used his translation of Plantamour's formula (§ 22).

§ 49. *Whitney's Method.*—"From observations made in connection with the Geological Survey of California, a series of corrections were deduced for reducing the barometric readings made at different hours of the day, of the different days of the different months, and for the different altitudes to the daily mean for the year. These corrections can only be used in the neighborhood in which the observations on which they were based were made. Similar tables made for different climates would differ materially from each other." For tables constructed upon this principle for the climates of Germany, Philadelphia, and Greenwich, respectively, see Guyot's Collection, Group D, 3d Ed., pp. 80, 81, 93, 94.

§ 50. *Plantamour's Method.* — In the hypsometric survey of Switzerland, Plantamour made simultaneous observations of the barometer, thermometer, and psychrometer at Geneva, St. Bernard, and at the station whose height was to be determined. The approximate difference of altitude between the new station and Geneva, and between it and St. Bernard, were computed by Plantamour's formula (§ 22); the difference of elevation between Geneva and St. Bernard was also computed. The computed difference of elevation between Geneva and St. Bernard compared with the actual difference of altitude, as determined by the spirit-level, gave a correction to be applied to the computed differences for the new stations. The ingenious details of the computation are too complex to be described here.

Marshall * and Rühlmann applied methods somewhat similar to the above.

§ 51. *Gilbert's Method.*—This method,

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* Wheeler's Geological Survey, Vol. II., p. 522.

which has already incidentally been fully described, is somewhat similar to the above, but differs from them in determining the height of the new station by the sole means of the observed pressures. A comparison between it and the several other methods seems to prove that it is the most accurate.*

§ 52. Unfortunately all methods of eliminating gradients involve considerable time and expense, and even then do not thoroughly accomplish the desired end, all of which shows that when great accuracy is desired the barometer should be dispensed with altogether, and the difference of elevation determined by some other means.

§ 53. *Sources of Error.*—The principal sources of error, as well as the means of eliminating them, have already incidentally been discussed, and need only to be referred to here.

1. *Instrumental Errors*, such as index error, imperfection of the scale, imper-

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* Gilbert's U. S. G. S. Report, 1881

fect adjustment for capillarity of the tube, impure mercury, and errors in the attached thermometer.

The first is usually eliminated by an adjustment of the zero of the scale, and with a good instrument the others would be inappreciable.

2. *Errors of Observation*, as inaccuracy of making contact between the ivory point and the mercury, inaccuracy of the reading itself, and the inaccuracy in determining the temperature of the barometer. Gilbert * from a comparison of 360 pairs of observations made by the Signa Service and the Geological Survey found the average error of observation to be a trifle less than three-thousandths of an inch. This difference does not involve the personal error between two observers, which, for even expert observers, may be nearly as much more.†

3. *Errors due to Gradient*, diurnal, annual, and abnormal, and those due to temperature and humidity. The errors

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* Report U. S. Geolog. Survey, 1880–1, p. 542.

† U. S. C. S. Report, 1870, p. 79.

of this class may have almost any value; the various methods of partially eliminating them have already been discussed.

4. *Errors due to the Effect of the Wind.*—The wind may cause either a condensation or rarefaction of the air in the room in which the barometer is, or even in the cistern of the barometer itself. This effect will vary with the velocity of the wind, with the position of the openings with reference to the wind, etc. On Mount Washington, a wind of 50 miles per hour caused the barometer to read .13 of an inch too low. Its effect will vary as the square of the velocity. It may be nearly, if not wholly, eliminated by having two apertures, one each on the windward and leeward side of the inclosed space.*

A similar effect of the wind is caused when the instrument is read in the immediate vicinity of any body which obstructs the wind. For example, if the barometer is observed on the windward side of a mountain, the reading will be

* Gilbert, U. S. Geolog. Survey Report, 1880–1, p. 562.

too high; if on the leeward, too low. The only way to avoid this difficulty is in the selection of the stations; but it is not always possible so to avoid it.

§ 54. *Limits of Precision.*—It is sometimes stated that "the barometer is the most accurate instrument for determining differences of level." It needs only a moment's reflection to see that this cannot be true. The following results, given by Professor Guyot, are frequently quoted as showing the great accuracy of barometric instruments:

Mont Blanc, by barometer, 15,781 feet;
by spirit-level, 15,780 feet.
Mount Washington, by barometer, 6,291.7 feet;
by spirit-level, 6,293 feet.
In North Carolina, by barometer, 6 701 feet;
by spirit-level, 6,711 feet.
In North Carolina, by barometer, 5,248 feet:
by spirit-level, 5,246 feet.

These results are to be considered exceptional, and only obtained by numerous repetitions in various states of the atmosphere.

The difference of altitude computed from one or even several days' observa-

tions can not be relied upon as being more than a rough approximation. This has been shown by Williamson,* who has computed the difference of altitude between Geneva and St. Bernard (using the same formula as in the last three examples quoted above) for every day for two years from daily simultaneous observations. The difference between the result by the barometer and the spirit-level, in several cases, was more than 3 per cent. Under less favorable circumstances the errors were even more than twice as great.†

The altitude computed by the monthly mean of daily observation for different months of the same year, and also for the same month of different years, differ as much as 1 per cent.‡

§ 55. The following differences between the results by the barometer and the spirit-level do not indicate that high degree of accuracy in barometric hypsometry, even where a long series of observa-

* Williamson on the Barometer, p. 206.

† See also Gilbert, p. 456–9.

‡ Williamson on the Barometer, p. 236.

tions is used, which was formerly supposed to be attainable by this means. The results by the barometer were obtained by computing the difference of altitude from monthly means of the mean of the daily observations, and taking the mean for the time stated.*

Sacramento and Summit,
3 years' observations, – 24 ft. in 6,989 ft.
Geneva and St. Bernard,
12 years' observations, – 2 6 met. in 2,070 met.
Portland and Mount Washington,
6 years' observations, + 37 ft. in 6,289 ft.
Vera Cruz and City of Mexico,
1 year's observations, + 5 met. in 2,282 met

§ 56. For an interesting comparison of the absolute, and also the relative, errors of the various methods, see Gilbert's Memoirs, Chapt. III, in U. S. Geological Report, 1880–1.

For additional data concerning the accuracy of barometric leveling, see U. S. C. & G. Survey Report, 1870, p. 88; do., 1871, p. 154–75; do., 1876, p. 355–76.

* U. S. C. S. Report, 1881, p. 254.

§ 57. Although the barometer can not be regarded as a hypsometric instrument of great precision, yet with care it can be made to give results with sufficient accuracy for reconnoissance or exploration. For this purpose it is unexcelled by any other instrument, but this is about the only use of the instrument to the engineering profession.

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## CHAPTER II.

### LEVELING WITH THE ANEROID BAROMETER.

#### ART. 1. THE COMMON ANEROID.

§ 58. *Description.*—The aneroid barometer consists of a cylindrical metallic box, exhausted of air, the top of which is made of thin corrugated metal, so elastic that it readily yields to alterations in the pressure of the atmosphere. When the pressure increases, the top is pressed inwards; when it decreases, the elasticity of the lid tends to move it in the opposite direction. These motions are transmitted by delicate multiplying levers to an index which moves over a scale. A spring is

sometimes inserted between the two ends of the vacuum chamber to reinforce the elasticity of the corrugated ends. Sometimes the vacuous box is not entirely exhausted, the object being that the enclosed air may reinforce the spring, "the air gaining elasticity as the spring loses, with increase of altitude." It is at least doubtful whether the spiral spring needs assistance, and therefore whether the air is of any benefit; and it certainly introduces complications, owing to the effect of a change of temperature of the enclosed air.

There are several forms of aneroids which differ in the mechanism employed to multiply the linear motion of the end of the vacuous box and to convert it into angular motion. Fig. 5 shows the mechanism of a common form; the outside case and the front face of the vacuous box are removed.

The instrument is graduated empirically by comparing its indications under different pressures with those of a mercurial barometer; the scale is marked to

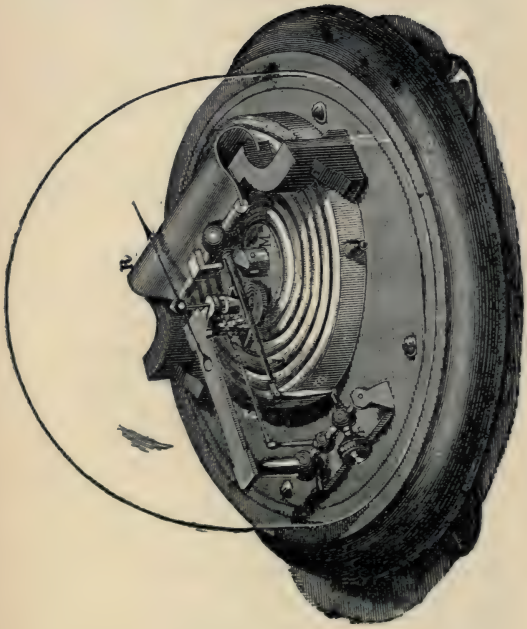

FIG. 5.

correspond to inches of the ordinary barometer column, the inches being divided into tenths, and the tenths usually into four parts. At the back of the instrument is a little screw which presses against one end of the exhausted box; by turning this screw the index can be moved over the scale, and the instrument may thus be made to agree at any time with a standard mercurial barometer.

§ 59. In many instruments there is an additional scale of altitudes in feet generally divided according to a table prepared for the purpose by Professor Airy. Such a table could be prepared by using any of the formulæ discussed in the preceding chapter by neglecting the corrections. Professor Airy used a formula similar to (11), and neglected the temperature term. When the aneroid has a scale of elevations engraved upon its face the approximate difference of height is obtained by subtracting the reading in feet at the lower station from that at the upper.

The use of such a scale leads only to

rough approximations, as it is based on the assumption that certain differences of pressure correspond at all heights with the same differences of elevation. The scale of elevations can only be correct at some particular temperature, and hence in general a temperature correction must be applied. Some makers endeavor to eliminate this correction by making the scale movable. "The movable scale is unscientific and inaccurate." The best plan is to dispense with scales of altitude, whether fixed or movable, and calculate the heights.

§ 60. *Defects.*—The aneroid is a very convenient instrument, and for a stationary instrument where nice readings are not required, it does very well; but for accurate hypsometrical results it is an inferior instrument. Its defects are:

1. The elasticity of the corrugated top of the vacuum chamber is affected by repeated changes in pressure. This will produce error in the scale readings.

2. It is usually claimed that, in consequence of not completely exhausting

the vacuum box, the indications of the aneroid become independent of the effect of changes of temperature of the instrument. The best that can be hoped for is that for small changes the temperature correction is less than the error of observation. In instruments compensated for temperature, the effect of a change is sometimes the same as that in the mercurial barometer and sometimes the reverse. The effect of the temperature on any particular instrument can be determined only by trial.

3. The different spaces on the scale are seldom correct relative to each other, owing probably to errors of observations and graduation, and possibly to differences of temperature and changes in elasticity. As a matter of fact, the scale is often only a scale of equal parts. The barometer scale is more accurate than the elevation scale, since the latter has all the inaccuracies due to the formula by which it is graduated, in addition to those of the instrument itself. For accurate work the aneroid should have a ther-

mometer attached; then, before using it, it should be tested under an air-pump, together with a mercurial column, and its scale errors for different temperature and pressures determined.

4. The weight of the machine affects its indication, *i.e.*, the reading of the aneroid will differ when held in different positions. "In the best instruments this difference is sometimes as much as 0.008 of an inch, corresponding to a difference of elevation of about 8 feet." (Williamson.)

5. Like all combinations of levers, screws, and springs, the aneroid is subject to continual shifting of parts, when subject to the jars and jolts encountered in transportation and in use. The only remedy is frequent comparisons with a mercurial barometer.

6. The aneroid is deficient in precision since the least reading is 0.025 of an inch, which corresponds nearly to 25 feet of elevation.

7. With most aneroids the spring ceases to act after the pressure has been lowered

somewhat; that is, the instrument runs down. Before using the instrument, experiments should be made to determine the range of pressure to which it may be exposed before the spring ceases to act. In case an aneroid is to be used in an elevated region, if there is a mercurial barometer with the party, screw up the aneroid until the spring acts well and set it by the mercurial barometer, so that there shall be a difference of say one or two inches between them.

§ 61. "With all these defects a good aneroid is of much assistance on a survey or reconnoissance in mountainous districts, on side trips of one or even several days' duration, when the instrument had been previously compared with a standard mercurial barometer at various temperatures and in different elevations and proper tables of corrections made. It is evidently important that there should be a good attached thermometer. It should be compared before and after it is used in that way, to see if the zero has not changed in the meantime, and if the

agreements are satisfactory the results can be relied upon." *

§ 62. *Formulæ.*—From the readings of the aneroid at two stations, the difference of elevation may be determined by any of the formulæ of the preceding chapter, for any of the approximate formulæ are as accurate as the instrument. A modification of Babinet's approximate formula (11) is most frequently used. The following is a very common form:

$$X = 54500 \frac{H_1 - H'}{H_1 + H'} + \frac{T-55}{450} \pm 10 \pm \frac{X}{200}. \dagger$$

The last term is the supposed probable error due to the varying density of the air column, and the preceding term that due to the instrument itself. This formula is limited to difference of heights of about 3,000 feet.

## Art. 2. The Goldschmidt Aneroid.

§ 63. The common aneroid was invented by Vidi, of Paris, in 1847, and the defects of its complex levers have long

* Williamson on the Barometer.

† U. S. C. S. Report, 1876, p. 352.

been recognized. As early as 1857, Goldschmidt designed a form of aneroid which

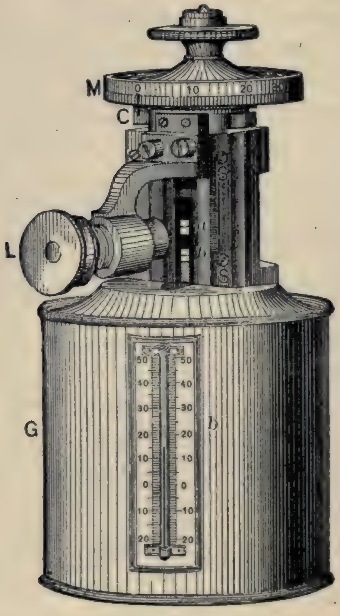


FIG. 6.

sought to do away with the transmitting and multiplying mechanism of the Vidi

form. Figs. 6 and 7 are two views of one of the latest forms of Goldschmidt's aneroids. Fig. 7 is a section through the com-

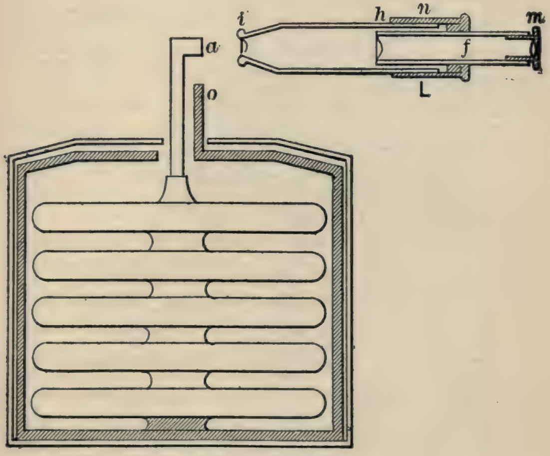


FIG. 7.

pound vacuum chamber; the greater the number of boxes the larger the motion of the index *a*. The relative position of the movable index *a* and fixed point of reference *b* is observed by the telescope L (Fig. 6), the distance being measured by

the micrometer M. The instrument is very delicate in its indications, but is liable to serious disarrangement by ordinary handling. Different manufactures have slightly different forms of the Goldschmidt type, but all have essentially the same defects—are not able to stand ordinary use.

It is doubtful if there is any advantage in an aneroid as complicated as that shown in Figs. 6 and 7; it seems probable that no form can be devised which shall be both delicate in its indications and able to stand rough handling. The chief advantage of the common aneroid is its portability, combined with moderate accuracy. The mercurial barometer and the aneroid supplement each other; the first is delicate and the second is portable. It is doubtful if the two qualities can be combined in a single instrument, or one obtained more delicate or more reliable than the mercurial barometer.

## Chapter III.

### LEVELING WITH THE THERMO-BAROMETER.

§ 64. *Theory.*—When water is heated, the elastic force of the vapor produced from it gradually increases until it becomes equal to the incumbent weight of the atmosphere. Then, the pressure of the atmosphere being overcome, the steam escapes rapidly in large bubbles and the water boils. Since the temperature at which water boils in the open air depends upon the weight of the atmosphere column above it, and as the weight of the atmosphere decreases with the elevation, it is obvious that the higher the station, the lower the temperature at which the water will boil at that station. The temperature at which water boils under different pressures has been determined by experiment. It is then only necessary to observe the temperature at each station at which water boils, and by referring

to tables similar to the above, find the corresponding height of the barometer, from which the difference of elevation may be computed by any of the formulæ previously given. Or the temperature

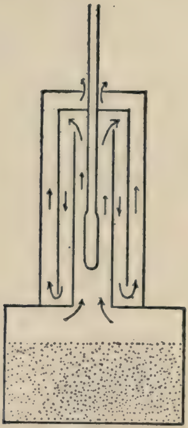

FIG. 8.

may be observed at only one point, and by using the mean pressure at the sea-level, compute the absolute elevation.

Or, finally, if the effect of variations in temperature, moisture, pressure, etc.,

be neglected, a table may be computed which will give, with the observed temperature of boiling water for an argument, the average approximate elevation of the station above the sea.

TABLE OF ELEVATIONS CORRESPONDING TO DIFFERENT TEMPERATURES OF BOILING WATER.

| Boiling-point. | Elevation above the sea | Boiling-point. | Elevation above the sea |
|---|---|---|---|
| 190° | 11,719 ft. | 208° | 2,049 ft. |
| 195 | 8,953 | 209 | 1,543 |
| 200 | 6,250 | 210 | 1,021 |
| 202 | 5,185 | 211 | 509 |
| 204 | 4,131 | 212 | 0 |
| 206 | 3,085 | 213 | −507 |

A table similar to the above, which depends upon a mean state of the atmosphere, can not be very reliable, but it is as accurate as the observations themselves.

§ 65. *Description.*—This instrument is very simple in its construction, requiring only some means of immersing a sensitive thermometer in the steam which arises

from pure water while boiling under atmospheric pressure.

The general arrangement consists of a closed vessel with a chimney with a combination of passageways for the exit of the steam, somewhat like Fig. 8. The bulb of the thermometer is thus immersed in a current of steam. The double passageway is to prevent condensation on the inner walls of the flue.

§ 66. *Defects.*—The chief difficulty is in ascertaining with the necessary accuracy the true temperature of boiling water; an error of $\frac{1}{10}$ of a degree centigrade would cause an error of 70 to 80 feet in the final result. An observation of the boiling-point, differing by $\frac{1}{10}$ of a degree from the true temperature, ought to be considered a good one. The accuracy is dependent upon the accuracy and sensitiveness of the thermometer, and is affected by the quality of the glass of the thermometer, the form and substance of the vessel containing the water, the purity of the water, the place at which the bulb of the thermometer is placed, whether in

the current of steam or in the water, the error of reading, displacement of zero point, etc.

Even if the above errors did not exist, this method would still be subject to all the chances of error which affect the measurements of heights by the barometer.

Nor is the thermo-barometer as convenient as either the aneroid or mercurial barometer, owing to the time required to start a fire, boil the water, make the observation, and wait for the instrument to cool, and the difficulty of obtaining pure water. Also, altogether the apparatus makes quite a load to be carried from place to place.

Since the invention of the aneroid the method of measuring heights by the temperature of boiling water has almost been abandoned.

## Chapter IV.

### TRIGONOMETRIC LEVELING.

§ 67. *Principle.* — Trigonometric leveling consists in determining the difference of level of two stations by means of the measured angle of elevation or zenith distance of one and the known horizontal distance between them. The horizontal distance is usually given by triangulation.

This kind of leveling is peculiarly suitable for finding the heights of the stations of a triangulation survey, since the extra labor required to measure the necessary vertical angles is but slight.

§ 68. *Observations.*—The vertical angles are measured at the same time and with the same instrument as the horizontal angles. The instrument should have two opposite verniers or micrometer microscopes and a sensitive level in a plane parallel to the vertical circle. It should be carefully adjusted for collimation, ver-

ticality of the vertical axis, horizontality of the horizontal axis of the telescope, and the verticality of the plane of the vertical circle. Matters relating to stations, towers, and targets belong more particularly to triangulation and will not be discussed here. On account of the uncertainty of the action of the atmosphere, there is no hope to obtain as great accuracy in the measuring of vertical as in the horizontal angles.

All errors of the instrument, except those of graduation, will be eliminated by observing as follows: Sight upon a target, read the level and the circle; reverse in altitude and azimuth, point upon the station and read circle and level again. The half difference of reading, corrected for difference of level, is the zenith distance of the target. Shifting the circle and repeating would reduce the errors of graduation and observation, but these errors are generally so small in comparison with the uncertainties arising from refraction that it is therefore better to measure the angles on different days, so

as to obtain different conditions of the atmosphere rather than to take any great number of successive observations.

§ 69. *Refraction.*—Since the effect of atmospheric refraction is to elevate objects on the horizon, a correction for refraction must be added to the zenith distance as measured above. The uncertainty as to the amount of this correction is the great cause of inaccurate work in trigonometrical leveling. Refraction is so erratic in its character that no satisfactory method has yet been devised for determining it. The best that can be done is to observe only when the refraction has its least effect.

Therefore, since the accuracy of trigonometrical leveling is limited by our knowledge of the laws of atmospheric refraction, it will be necessary to investigate that branch before proceeding with the general subject.

## ART. I. COEFFICIENT OF REFRACTION.

§ 70. *Definition.*—The angle of refraction divided by the arc of the earth's

circumference, intercepted between the observer and the station observed, is called the *coefficient of refraction.** That is, if C = the angle at the center of the earth subtended by the two stations, F = the refraction angle and $m$ = the coefficient of refraction, then $m = \frac{F}{C}$. C can be found in arc from the expression $C'' = \frac{k}{\rho \sin 1''}$, in which $k$ is the distance between the two stations and $\rho$ is the radius of the earth.

§ 71. *To Find the Coefficient.*—The coefficient of refraction may be found in either of two ways, viz.: I, from reciprocal zenith distances; or, II, from the observed zenith distances of two stations, the relative altitudes of which have been determined by the spirit-level.

§ 72. I, *By Reciprocal Zenith Distances.*—In Fig. 9, if A and B denote the posi-

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* In the reports of progress of the U. S. Lake Survey and in Wright's Adjustment of Observations a definition is given which makes the coefficient of refraction twice as large as the above. The definition as above seems to be the one more frequently used.

tions of the two stations, the angles which the observers attempt to measure are PAB and QBA. On account of the re-

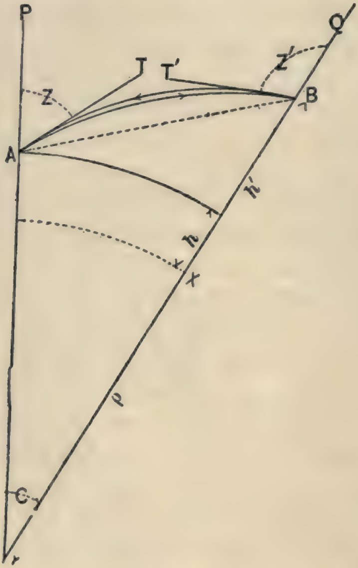


Fig. 9.

fraction of the atmosphere, the path of a ray of light from B to A will not be a straight line, but some curve more or less

irregular; the direction in which B is seen from A will be that of a tangent, AT, to this curve. The line of sight from the other station will not necessarily be over the same curve.

Let Z and Z′ be the observed zenith distances and $m$ and $m'$ the respective coefficients of refraction; $m$C and $m'$C are the refraction angles. From the figure

$$Z + mC + Z' + m'C - C = 180° \qquad (19)$$

Since this equation contains two unknown quantities, it shows that observations over a single line will not give the coefficients; but in the reduction of a triangulation net, this equation may be applied to each line and the resulting equation solved by the method of least squares, thus obtaining the values of each coefficient, or at least a coefficient for each station.

However, such elaboration is of doubtful utility and is seldom practiced. It is usual to assume that the mean of a number of observations, taken under favorable conditions, will eliminate the difference of

refraction which is found to exist, even at the same moment, at two stations a few miles apart. The observations should be simultaneous or nearly so. Under these conditions the coefficients at the two ends of a line become equal; *i.e.*, in the preceding formulæ $m'=m$. The error in this assumption will be greater as the distance between the stations is greater; it also increases very rapidly with the difference in elevation.

Making this substitution (19) becomes

$$m=\tfrac{1}{2}\left(1-\frac{Z+Z'-180}{C}\right)* \qquad (20)$$

§ 73. II, *By zenith distances, the difference of level being known.*—There are two cases: (1) single zenith distances and (2) reciprocal zenith distances.

(1) Measure the zenith distance of the station, and from the known difference of level compute the true zenith distance; the difference between the true and computed zenith distances is the refraction angle, which, divided by the subtended angle, is the coefficient sought.

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* U. S. C. & G. Report, 1882, p. 182.

From Fig. 9,

$$\frac{\mathrm{CB}-\mathrm{AC}}{\mathrm{CB}+\mathrm{AC}}=\frac{h'-h}{2\rho+h'+h}=\frac{\tan.\ \frac{1}{2}(\mathrm{CAB}-\mathrm{CBA})}{\tan.\ \frac{1}{2}(\mathrm{CAB}+\mathrm{CBA})} \qquad (21)$$

$$\mathrm{CAB}=180^\circ-z-mc,$$
$$\mathrm{CBA}=z+mc-c.$$

Substituting and reducing,

$$\frac{h'-h}{2\rho+h'+h}=\cot.\ (z+m\mathrm{C}-\tfrac{1}{2}\mathrm{C})\ \tan.\ \tfrac{1}{2}\mathrm{C} \qquad (22)$$

Developing tan. $\frac{1}{2}$C by

$$\tan.\ x=x+\frac{x^3}{3}+\text{ etc.},$$

and substituting $\mathrm{C}=\frac{k}{\rho}$, we get

$$h'-h=k\cot.\ (\mathrm{Z}+m\mathrm{C}-\tfrac{1}{2}\mathrm{C})\left(1+\frac{h'+h}{2\rho}+\frac{k^2}{12\rho^2}\right) \qquad (23)$$

$$\mathrm{Z}+m\mathrm{C}-\tfrac{1}{2}=\cot.^{-1}\frac{h'-h}{k}\left(1-\frac{h'-h}{2\rho}-\frac{k^2}{12\rho^2}\right) \qquad (24)$$

from which $m$ can be found.

(2) Also $\mathrm{CBA}=180^\circ-(\mathrm{Z}'+m'\mathrm{C})$; substituting this value in (21), remembering

that $Z' + m'C = Z'_0$ and $Z + mC = Z_0$ and solving as before we get

$$\tfrac{1}{2}(Z'_0 + Z_0) = \tan.^{-1}\frac{h'-h}{k}$$

$$\left(1 - \frac{h'-h}{2\rho} - \frac{k^2}{12\rho^2}\right)^* \quad (25)$$

$$\tfrac{1}{2}(Z'_0 - Z_0) = 90° + \tfrac{1}{2}C. \quad (26)$$

$$Z_0 - Z = F \text{ and } Z'_0 - Z' = F'$$

and finally $m = \frac{F}{C}$ and $m' = \frac{F'}{C}$.

Struve,† Bauerfiend, and others have deduced rational formulas for computing the coefficient of refraction from the observed temperature and barometric pressure, but such formulas are of little utility, owing to the difficulty in the way of getting the temperature and pressure of the atmosphere.

§ 74. *Laws of Refraction.* — "Experience has proved that the refraction is greater and more variable at sunrise than at any other hour of the day; that it gradually diminishes in both respects,

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* U. S. C. & G. Report, 1882, p. 183.
† British Ordnance Survey, p. 214.

until 9 or 10 A.M.; that between those hours and $3\frac{1}{2}$ P.M. it is comparatively stationary, and from $3\frac{1}{2}$ P.M. to sunset it increases in amount and variation, being the greatest during the night. The best period for observing, therefore, is between 9 A.M. and $3\frac{1}{2}$ P.M., and the worst at sunrise and sunset." *

During the night the refraction is less variable, but greater in amount, one about offsetting the other. A day with the sky wholly overcast is to be preferred to a clear or partially clear day.

"Although the refraction exhibits daily variations, and is a function of the temperature and the pressure of the atmosphere, yet it is extremely irregular; in its ordinary variations the coefficient keeps within the range of $\frac{1}{9}$ to $\frac{1}{16}$, but occasionally and abnormally it may be several times greater, or it may become zero, or even take a negative value. The refraction is slightly greater for lines crossing water than for lines over land; it diminishes with altitude and with increasing

* U. S. C. S. Report.

temperature, but increases with increasing atmospheric pressure; in general its value depends upon the law of the distribution of temperature with the height." *

There is an irregular effect of refraction, usually termed "boiling," due to varying density of the atmosphere, which causes the target to vibrate rapidly through a small angle rarely exceeding 10″, but whose effect cannot be calculated.

For a table showing the diurnal variations of refraction see U. S. C. & G. Report, 1876, pp. 361–2; also U. S. C. & G. Report, 1883, pp. 295 and 310.

§ 75. *Value of the Coefficient of Refraction.*

| | | |
|---|---|---|
| De Lambre, from observations in France | 0.07876 | —1 |
| Bégat's Traité de Geódésie, in France, for Summer | 0.06 | —2 |
| Bégat's Traité de Geódésie, in France, for Winter | 0.10 | —2 |
| Bégat's Traité de Geódésie, in France, for mean | 0.08 | —2 |

* U. S. C. S. Report.

| | | |
|---|---|---|
| Bessel, from operations in Prussia.. | 0.0685 | —3 |
| Gauss, " " " " | 0.0653 | —3 |
| Struve, " " " Russia... | 0.0618 | —3 |
| Corabeuf, " " " " | 0.0642 | —4 |
| Mean from all observations in France.......................... | 0.0665 | —3 |
| Chauvanet gives................. | 0.0784 | —1 |
| The British Ord. Survey.. .0.0768± | 0.0035 | —6 |
| Do. for rays not crossing the sea.... | 0.0750 | —6 |
| " " " crossing " " | 0.0809 | —6 |
| U. S. C. S. in New England near the sea........................... | 0.078 | —5 |
| U. S. C. S. in New England for small elevations................ | 0.075 | —5 |
| U. S. C. S. in New England between primary stations............... | 0.071 | —5 |
| U. S. C. S. in interior of country.... | 0.065 | —7 |
| U. S. Lake Survey in Central Illinois | 0.06 | —4 |

1. Chauvenet's Practical Astronomy, Vol. 1, p. 177.
2. Davies and Peck's Mathematical Dictionary, p. 20.
3. Ordnance Survey, p. 512.
4. U. S. Lake Survey Report, p. 428.
5. U. S. C. R. 1868.
6. Clarke's Geodesy, p. 283.
7. U. S. C. S. 1868, p. 62.

## ART. 2. TRIGONOMETRICAL LEVELING.

§ 76. *By Observed Angle of Elevation.*—Let D, Fig. 10, represent the position of the observer, and E the station whose

elevation is to be determined. Let A= the observed angle expressed in seconds of arc; $k$=the distance between the stations.

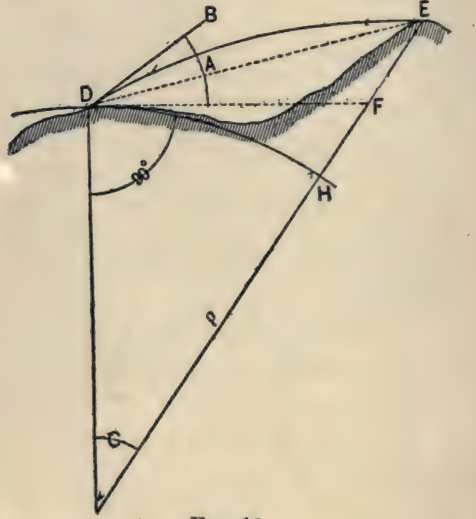


FIG. 10.

The difference of level, $dh$, is made up of HF, due to curvature, and FE, due to the angle of elevation. The angle of refraction $\mathrm{EDB}=m\mathrm{C}=m\frac{k}{\rho \text{ arc } 1''}$. The true

angle of elevation $EDF = A - \frac{km}{\rho \text{ arc } 1''}$.

$$FE = k \tan.\left(A - \frac{mk}{\rho \text{ arc } 1''}\right) =$$

$$k \tan. 1''\left(A - \frac{mk}{\rho \text{ arc } 1''}\right)''$$

$$FH = \frac{k^2}{2\rho}.$$

$$\therefore dh = EH = FE \pm HF =$$

$$kA \tan. 1'' - \frac{mk^2}{\rho} \pm \frac{k^2}{2\rho}. \quad (27)$$

The U. S. Coast Survey Report for 1868, page 127, states in effect that the error produced by neglecting the term $\frac{mk^2}{\rho}$ is no greater than the uncertainty in the coefficient of refraction, and gives the following as being applicable for distances not exceeding 10 or 15 miles:

$$dh = 0.00000485\ kA \pm 0.0000000667\ k^2 \quad (28)$$

The neglected term is $0.1k^2$ ft. for $k$ in miles. If this term is not neglected

$$dh = 0.00000485\ kA - 0.0000001334\ mk^2$$
$$\pm 0.0000000667\ k^2. \quad (29)$$

The last term is plus for angles of elevation. The chief source of error in this as in all other formulas for trigonometrical leveling is the uncertainty in $m$.

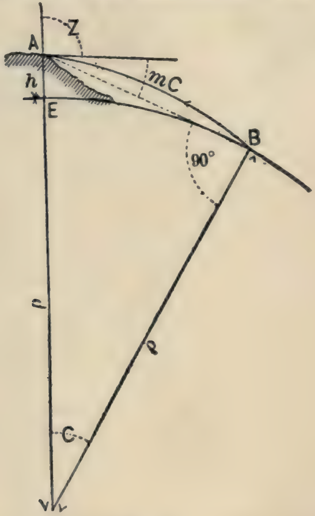


Fig. 11.

§ 77. *By the observed zenith distance of the sea horizon.*—Let $Z$ = the measured zenith distance, $h$ = the elevation sought. Then in Fig. 11

$$\cos. C = \frac{\rho}{\rho + h}. \quad \text{Hence } h = \rho \frac{1 - \cos. C}{\cos. C}$$

$$= \frac{2\rho \sin.^2 \frac{1}{2}C}{\cos. C} = \frac{2\rho \sin. \frac{1}{2}C \sin. C}{2 \cos. \frac{1}{2}C \cos. C} =$$

$$\rho \tan. \tfrac{1}{2}C \tan. C.$$

Since C is always small, assume $\tan. \frac{1}{2}C = \frac{1}{2} \tan. C$, which, substituted above, gives

$$h = \tfrac{1}{2}\rho \tan.^2 C.$$

To find C, notice that $Z + mC + (90 - C) = 180°$, therefore $C = \frac{Z - 90°}{1 - m}$, which, substituted in the above, gives

$$h = \tfrac{1}{2}\rho \tan.^2 \frac{Z - 90°}{1 - m} = \frac{\rho}{2(1 - m)^2} \tan.^2 (Z - 90).* \quad (30)$$

§ 78. *By the zenith distance observed at one station.*—Let $dh$ = the difference in level between the two stations. The triangle ABD, Fig. 12, gives $dh = k\frac{\sin. BAD}{\sin. ABD}$

$$EAB = mC = FBA = m'C.$$

$$ABD = Z + mC - C$$

$$BAD = 90° - (Z + mC) + \tfrac{1}{2}C.$$

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* U. S. C. S. Report, 1868, p. 127.

$$dh = k\frac{\cos.\ (Z+mC-\tfrac{1}{2}C)}{\sin.\ (Z+mC-C)}.^{*} \qquad (31)$$

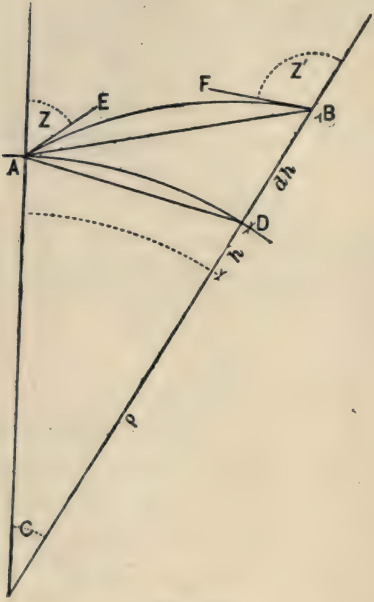


FIG. 12.

§ 79. *By reciprocal zenith distances to eliminate refraction.*—Let Z and Z′ be

* U. S. C. S. Report, 1868, p. 126.

the measured zenith distances. Then the triangle ABD, Fig. 12, gives

$$k : \sin. ABD :: DB : \sin. DAB.$$

Substituting for ABD and DAB their values in terms of $Z'$ add $Z$, assuming $mC = m'C$, and solving, gives

$$dh = DB = k \frac{\sin. \frac{1}{2}(Z' - Z)^*}{\cos. \frac{1}{2}(Z' - Z + C)}.$$

§ 80. *Micrometrical Difference of Level.*—Another method is to measure, by means of a micrometer inserted in the eye-piece of the telescope, the difference in altitude between the different stations, the absolute elevations of one or more being known. The method of reducing the observations is easily understood.

§ 81. *Corrections.*—In the preceding it has been assumed that the instrument and the target observed upon occupied what has been called the "station." The difference in height between the horizontal axis of the telescope and the trigonometrical point, or the ground at the station, and also the elevation of the target,

* U. S. C. S. Report, 1868, p. 125.

should be measured and made a part of the record.

The correction to the zenith distance for the difference in height of the target and telescope above the ground may be computed by the formula: correction in seconds $= \frac{d}{k \sin. 1''}$, in which $d$ is the difference in height and $k$ is the distance between stations.

§ 82. *Limits of Precision.*—Reciprocal zenith distances measured at any two stations at the same moment of time, or under the same supposed condition of atmosphere, give the best results. When reciprocal, but not simultaneous, the observations should be made on different days, as in the case of horizontal angles, in order to obtain as far as possible a mean value of the difference between the respective angles and an average value of the refraction. The same care should be taken when the zenith distance is measured at one station only.

The condition of the atmosphere and the relative refraction may be so different

at stations situated more than twenty miles apart, that, as a general rule, the difference of level determined even by reciprocal observations cannot be relied upon for the desired degree of accuracy at distances greater than about twenty miles, unless a very large number of measurements have been made, under the most favorable circumstances. Notice that the difference of height determined by trigonometrical leveling depends upon the coefficient multiplied by the square of the distance, and that, therefore, there is a limit to the distance for which any assumed mean coefficient can be depended upon for accurate results. The higher the elevations, the more reliable the results.

§ 83. In the final report of the U. S. Lake Survey (p. 544) it is stated that the probable error of determining a difference of level in Eastern Illinois by reciprocal zenith distances, using 8 to 10 separate measurements, made on 2 to 6 days, over lines 16 to 20 miles long, was somewhat less than 1 foot.

The U. S. C. & G. S. Report, 1876, p. 345, contains an account of trigonometrical leveling in California, over a line 14 miles long, in which the mean probable error of reciprocal zenith distances, 11 hourly observations on 5 successive days, was 0.2 meter in 600 meters; for single zenith distances the error was 1.09 meters. The same volume contains an account of angular leveling in Georgia, in which (p. 379) it is stated that the probable error per line of about 20 miles is .487 meter or $\frac{1}{68000}$ of the horizontal distance.

Probably the above may be considered as examples of the best work possible.

## Chapter V.

### Spirit Leveling.

§ 84. Spirit leveling may be divided into two divisions, viz.: ordinary leveling, or that undertaken, as a part of railroad surveys, drainage, etc.; and geodesic leveling, or that undertaken in connection with a triangulation survey, and in which extreme accuracy is sought. The latter is frequently called "precise leveling." The methods of the former are too well known to require discussion here; this chapter will be devoted exclusively to geodesic leveling.

### Geodesic Leveling.

#### Art. 1. The Instrument.

§ 85. Spirit leveling instruments may be grouped in three classes. The first includes all instruments that can be adjusted by reversals, the wye (or Y) level being a representative of this class. The second includes all that can not be adjusted by reversals, of which the "dumpy"

or English instrument is a type. The third includes all instruments whose errors of adjustment can be wholly eliminated by a system of double observations. The levels employed in geodesic leveling are of this class and are generally known as levels of precision.

There is considerable variety in the form of this class of levels, but only two have been used to any considerable extent in this country—the Swiss or Kern level, by the Lake Survey and Mississippi River Commission, and a modification of the Vienna or Stampfer level, by the Coast and Geodetic Survey. The construction of the two are similar, hence only the latter will be described here. The former is described in the final report of the U. S. Lake Survey, p. 597, and described and illustrated in Jordon's Vermessungskunde, vol. I., p. 404.

§ 86. *U. S. Coast Survey Level.*—The instrument is shown in Fig. 13.* The telescope may be reversed end for end and revolved about its optical axis, the

* From U. S. C. S. Report, 1879, p. 202.

two positions in which the horizontal thread is horizontal, being definitely fixed by projecting pins. The level can also be reversed end for end independent of the telescope. One end of the telescope and level can be raised and lowered by the micrometer screw. Near the micrometer is a cam-hook, by which the weight of the superstructure can be raised off the micrometer during transportation. Under the telescope are two false wyes on lever arms by which the telescope can be raised out of the wyes for transportation. The whole instrument is secured to the tripod head by a brass plate which fits over the feet of the leveling screws.

The aperture of the telescope is 43 mm. ($1\frac{3}{4}$ inches nearly), focal length 410 mm. ($16\frac{1}{2}$ inches nearly), magnifying power 37. The value of one millimeter of the level scale is $1''.5$ (1 inch $= 37''.5$ or radius $= 800$ feet). The diaphragm is glass and has two horizontal and one vertical line ruled upon it. The two horizontal lines are used as stadia hairs to determine the length of sight.

A smaller size of this instrument is also used. The weight of the larger one, inclusive of tripod, is 45 lbs.; the smaller weighs 23 lbs.

§ 87. *The Rod.*—The rod used with this instrument was a target rod made of pine, 3″ by 1″, stiffened by a strip fastened along the middle of each face. One edge of the main strip has a self-reading graduation to cm. upon it. A brass strip, graduated to cm., is let into the side of the strip; it extends the whole length of the rod, being fastened immovably at the middle. Its temperature is determined by a thermometer let into the side of the wood. The target, which is moved by an endless chain, carries a short ivory scale, graduated to mm., which slides over the brass strip; the rod therefore reads to millimeters. The foot of the rod is a rounded piece of brass resting in a corresponding depression in the foot plate.* A handle and a disk level enable the rodman to keep the rod vertical.

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* Would it not be better if the end of the rod were hollowed out to fit a projection on the foot plate?

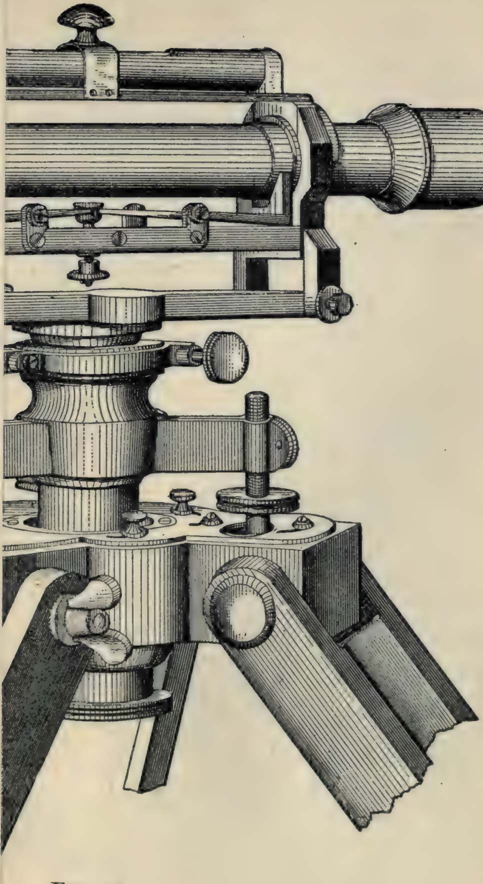

Fig. 13.

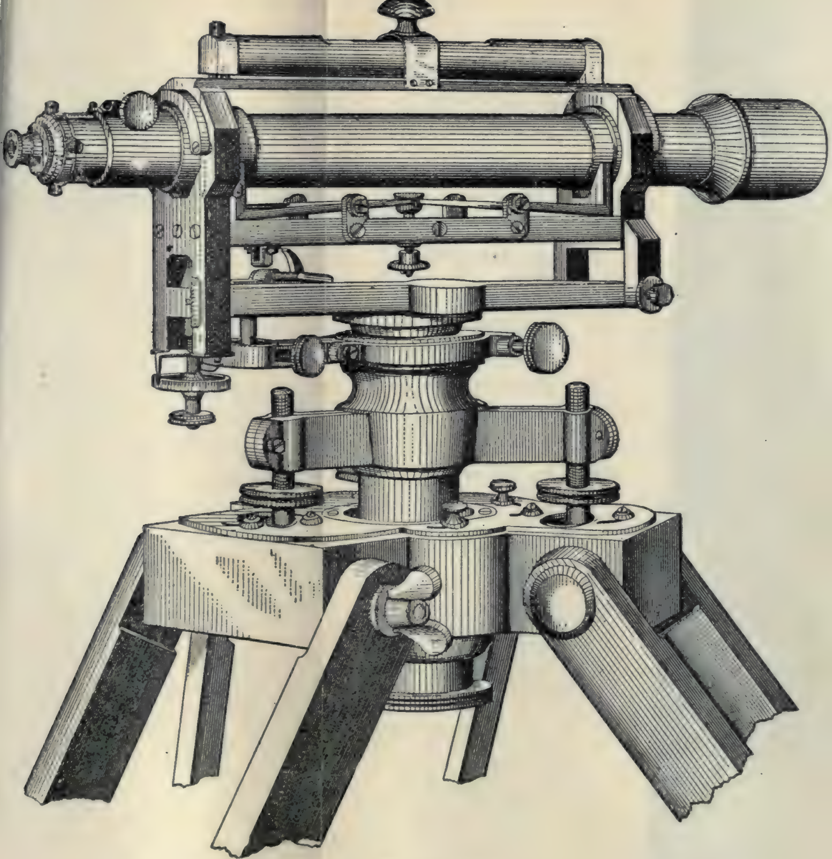

FIG. 13.

§ 88. *Adjustments.*—The adjustments of this or other geodesic levels do not materially differ from those of the ordinary forms, and are generally more conveniently made. In the ordinary forms some of the less important adjustments are generally neglected as being less than the degree of accuracy aimed at, but in precise leveling, extreme accuracy is desired, and therefore every adjustment must be carefully attended to, even though it is the intention to use the instrument in such a manner as to eliminate all errors of adjustment. The usual method is to adjust the instrument as nearly as possible and then determine the error; each single observation can then be corrected, thus affording a check between the members of a double observation.

§ 89. Inequality of diameter of collars is the only error that can not be eliminated by any system of double observations; it could be done if the line of the vertical axis were immovable; it is eliminated from the final result by equal

back and foresights. But to employ the check of double leveling (as will be described presently), a correction for inequality must be applied to the rod reading.

The inequality of the collars can be determined by observations with a striding level, exactly as the pivots of an astronomical transit are examined; consequently it is unnecessary to describe the method here (see Chavenet's Practical Astronomy, vol. II., p. 153, or U. S. C. & G. S. Report, 1880, p. 210). The correction to be applied to the rod reading at the distance D will be $d$ sin. $1''$, in which $d$ is the pivot correction in seconds of arc.

§ 90. The absolute value, and the uniformity of the graduations of the rod, the coefficient of expansion, the accuracy of the thermometer, and the disk level should be tested carefully. The observer should also know his ordinary inaccuracy in performing the different parts of an observation.

§ 91. *Correction for Curvature and Refraction. Curvature.*—To compute the

correction for curvature, let AD, Fig. 14, represent the line of apparent level, and AB the true level. DB is the correction for curvature. By Geometry $AD^2 = DB(2BC + DB)$. Neglecting DB,

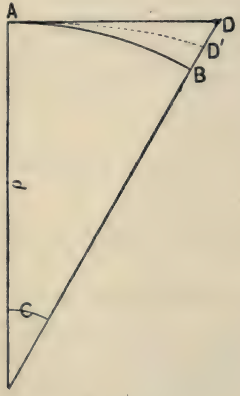


FIG. 14.

as it is very small in comparison with 2BC, and representing the length of sight by $k$, and the radius of curvature by $\rho$, then correction for curvature $BD = \frac{k^2}{2\rho}$.

For BD in feet, and the distance in miles thus becomes

$$BD = 0.667\ k^2.$$

*Refraction.*—D′ is the true position of the target and D the apparent. It has been shown (Art. 1, Chap. IV) that the angle of refraction D′AD is equal to $m$C, in which $m$ is the coefficient of refraction and C the angle ACD.

$$C'' = \frac{k}{\rho \sin. 1''} = k \tan. DAD'$$

$$DD' = k \tan. DAD' = k DAD' \tan. 1'' = \frac{mk^2}{\rho}.$$

It will be impossible to select a value of $m$ which shall be true for all cases; but since the line of sight is always near the ground, and since the observations are generally made in the morning and evening, when the seeing is best, a large value should be chosen. The Coast Survey uses * $m = 0.070$.

*Total Correction.*—The correction for curvature and refraction to be applied to

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* U. S. C. S. Report, 1882, p. 177.

the observed readings is

$$BD' = BD - DD' = (1 - 2\ m.)\ \frac{k^2}{2\rho} = \frac{.43k^2}{\rho}$$
$$= .000000020\ 45\ k^2.$$

Numerous tables have been computed which give this correction directly for the different lengths of sight; or tables can be prepared which will give the difference of the correction with the difference in length of sight for an argument.*

For $k = 210$ ft., the above correction $= .001$ ft.

For $k = 1$ mile, the above correction $= 0.57$ ft.

## Art. 2. The Practice.

§ 92. *Method.*—There are two principal methods in leveling, according to the sequence of the instrument and rod, which may be called *single* and *double* leveling. Each may be performed with one rod or with two; this gives rise to four methods, which are represented graphically in Fig. 15. $I_1$, $I_2$, $I_3$, etc., indicate successive positions of the instru-

* U. S. C. S. Report, 1882, p. 178.

ment; $A_1$, $A_2$, etc., successive positions of one rod; and $B_1$, $B_2$, etc., successive positions of the second rod.

I is the method of ordinary leveling, and may be called *single leveling with one rod.*

II is *single leveling with two rods.* By the use of the two rods, less time intervenes between the backsight and foresight; it is therefore more accurate, as there is not so much liability of change in the plane of the line of sight. It is also more rapid than with a single rod. This is the method used on the U. S. Lake Survey,* and the Mississippi River Surveys.†

III, *double leveling with one rod* affords a perfect check against errors of adjustment and observation, since the difference in reading of the two foresights should be the same as the difference of the two backsights following.

IV, *double leveling with two rods* combines all the advantages of II and III.

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* Chief Engineer's Report, U. S. A., 1880, p. 2427.
† Mississippi River Commission Report, 1881.

It is the method used on the U. S. C. & G. Survey.* The numerals adjacent to the rods show the order in which they are sighted upon.

V, *duplicate leveling*, is another method sometimes used. Two rods are used, being placed as in the figure. After having observed upon $A_1$ and $B_1$ the instrument is pulled up and reset a little to one side and the two rods sighted upon again. This method duplicates the work as far as instrumental errors are concerned, but is not an independent check.

§ 93. *The Observation.*—After having planted the tripod firmly and leveled the instrument, read both ends of the bubble, estimating the fraction of a division; next read the position of the two (or three) wires on the rod; then read the bubble again for a check to eliminate any change. Reverse the level end for end, and turn the telescope 180° about its optical axis, and repeat the operations as above. The first reversal eliminates any inequality in the lengths of legs of

* U. S. C. & G. S. Report, 1879, p. 206.

the striding level; the second eliminates any error of collimation. The mean of the several readings must be corrected for the difference in position of the bubble, and for inequality of the collars.

Instead of reversing the level and telescope at the same time, the observations are sometimes made as follows: Read upon the rod, reverse the level, and read again; reverse the telescope and read a third time, then reverse the level and make a fourth reading. The first method seems the better.

§ 94. If there is a milled head screw under one end of the telescope, the bubble can easily be brought to the middle each time; if there is not, it is better to bring it nearly to the middle and apply a correction. It is not enough to read only one end, since the bubble is liable to change its length with a change of position or temperature.

§ 95. On the coast survey the method of observing differs slightly from that described above.* Errors of level and

* U. S. C. & G. S. Report, 1879, p. 206.

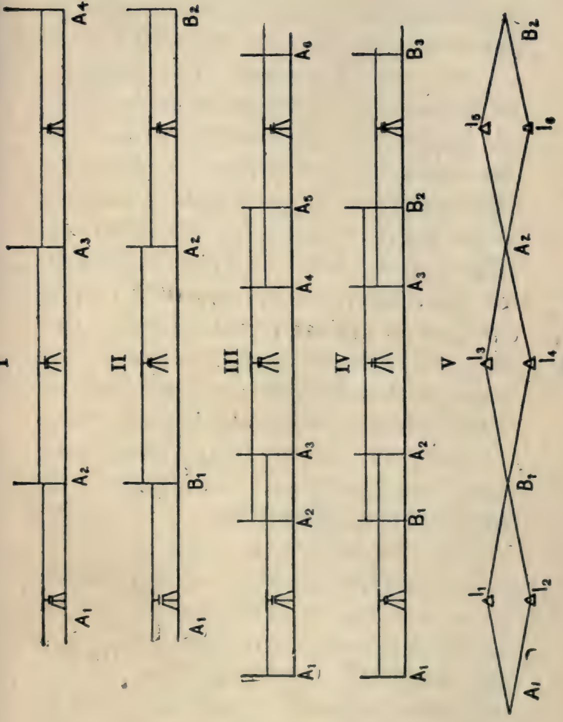
I
A1
A2
A3
A4
II
A1
B1
A2
B2
III
A1
A2
A3
A4
A5
A6
IV
A1
B1
A2
A3
B2
B3
V
A1
l1
l2
B1
l3
l4
A2
l5
l6
B2

FIG. 15.

collimation are eliminated by reversing bubble and telescope on each backsight and foresight; but each observation is of a single wire on a target. The target is set but once for each station, the differential quantities being read by the micrometer under the eye end of the telescope. This seems not to be as good a method as the above; "there are two objections aside from the time and labor required to set the target. First, there is no sufficient check against errors in reading the positions of the target. Second, the micrometer is read for a central position of the bubble, the telescope is then moved to bisect the target and the screw read again, therefore there is no check on the stability of the instrument."

§ 96. *Length of Sight.*—On the U. S. C. & G. Survey, the length of sight ranges from 50 to 150 meters, according to condition of ground and weather, the average being 110 meters, the distance between the two rods on the same side of the instrument being 20 meters. On the Lake Survey,* the maximum was 100

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* Final Report, p. 598.

meters; on the Prussian Land Survey, since 1879, it has not exceeded 50 meters, except for river crossings.*

The attempt is always made to place the instrument half way between the two stations; the rodman approximates the distance by stepping, and the instrument man measures it by the stadia hairs. On the Lake Survey, the difference between corresponding back and foresights was not allowed to exceed 10 meters.

§ 97. *Sources of Error.*—Probably in no other kind of instrumental engineering is it as important to distinguish between compensating and cumulative errors, as in leveling. For convenience in discussing them, we shall classify errors of leveling as follows: 1, Instrumental Errors; 2, Rod Errors; 3, Errors of Observation; and 4th, Personal Errors.

1. *Instrumental Errors.*—The principal instrumental error is due to the line of sight not being parallel to the level, which may be caused by imperfect adjustment, or unequal size of rings, or

* Wright's Adjustment of Observations, p. 375.

both. If the telescope slide is not straight, or does not fit snug, it will also produce an error. Of course the instrument must be focused so as to eliminate parallax. All of these errors are eliminated, whatever their value, by setting the instrument midway between the turning points.

It has been found that appreciable errors are caused by the settling of the instrument on its vertical axis, and of the settling of the tripod legs into the ground; in spongy or clayey ground the tripod legs are sometimes gradually lifted up. These errors, though small in themselves, are more important than is generally supposed, inasmuch as they are cumulative; but probably they would be appreciable only in precise leveling. They can be eliminated by running the line in the opposite direction.

A small source of error arises from the fact that the adhesion of the liquid to the sides of the glass tube prevents the bubble from coming precisely to its true

point of equilibrium. Even though it may finally arrive at the true point, it is liable to be read before it has stopped moving. Consequently a tube should contain considerable liquid, so as to give mass sufficient to overcome the adhesion.

Another small error is the effect of the sun in raising one end of the telescope by the unequal expansion of the different parts of the instrument. In ordinary leveling operations, the bubble is first brought to the middle and then the target is sighted in, leaving an interval for the sun to act. The error is greatest in working toward or from the sun. It is cumulative; for, on the backsight, one wye is expanded, which elevates the line of sight; while on the foresight the other wye is expanded, which depresses the line of sight, the two errors affecting the difference of elevation in the same way. The error on the foresight is farther increased by the cooling of the wye which was expanded during the backsight. The error due to the sun is always small, and can be nearly elim-

inated by noticing the position of the bubble after setting the target, and can be still further reduced by shading the instrument. The Indian Geodetic Survey proved conclusively that the error was appreciable, even when the instrument was shaded.*

2. *Rod Errors.*—The principal rod error is in not holding the rod vertical; this may be remedied by attaching a level or by waving the rod. In waving the rod, care must be taken that the front face is not lifted by resting upon the back edge when the rod is revolved backward.

With telescoping target rods when extended, the slipping of the upper piece, after the target has been pronounced correct and before the vernier has been read, is a source of error. The target itself may slip, but this is not so probable, because of its less weight.

Another source of error is the settling of the turning point, due, in coarse or sandy soil, to its own weight, or to the impact of setting the rod upon it. The resulting

* Jackson's Aid to Survey Practice, p. 181.

error is cumulative. The remedy in the first case is to use a long peg, or to rest the rod upon a triangular plate with the corners turned down slightly, or with spikes on the under side. This foot-plate with a convex button attached, on which to place the rod, is better than a peg for all cases. Whatever the turning point, the rod should never be dropped upon it.

Finally, another small rod error is the error in the graduated length. This affects only the total difference of elevation between the two points. This is a much more important source of error with the numerous home-made self-reading rods now in use, than with the rods made by regular instrument makers.

"An important source of error in spirit-leveling, and one very commonly overlooked, is the change in the length of the leveling rod from variations of temperature. It is quite possible that errors from this source may largely exceed the errors arising from the leveling itself." *

* Wright's Adjustments of Observations, p. 372.

3. *Errors of Observations.*—The principal error of observation is in reading the position of the bubble; even if the bubble is kept in the middle, it is nevertheless read. Every leveler should know the error on the rod corresponding to a given distance of reading of the bubble; he then knows how accurately he must read the bubble for a given degree of accuracy in the results.

Another source of error is the moving of the bubble after being read, and before the sighting has been made. This movement of the bubble may be caused by its being read before it has come to rest, by disturbing the instrument by stepping near the tripod legs, by turning the instrument slightly in azimuth, or by raising or lowering one end of the telescope in focusing, or by the action of the sun or wind. The bubble should be re-read after the target is nearly adjusted, or with a self-reading rod, after the reading has been made and before the rodman is signaled to move on. These two probably constitute the chief sources of error in leveling operations.

If the reading is made on either side of the vertical hair, there is a possibility of error, owing to the horizontal hair not being horizontal; with Y levels, not provided with means of preventing the rotation of the telescope in the Y's, this possibility becomes a probability. Any device which insures the horizontality of the horizontal hair increases the rapidity and accuracy of the work.

The inaccuracy of telling when the hair exactly covers the center of the target is a source of slight error, but not so small as many think. Owing to this source of error, the difference in accuracy between a target rod and a self-reading rod is not so great as their difference in precision. Because a target is read to thousandths is no evidence that it is accurate to that limit. In precise leveling, self-reading rods are generally used; to reduce the error of reading the position of the hair upon the rod, two or three horizontal hairs are used, the number depending upon the relative errors due to reading the target and the bubble. A reading is taken for

each hair and the mean used as the rod reading. For a single observation, a target rod is more accurate than a self-reading one; but three observations as above are probably more accurate than a single observation upon a target, and can be made in about the same time.

When very long sights are required to be taken with the level, another source of error must be considered, viz., the curvature of the earth and refraction. Owing to these two causes, a point 220 feet distant appears about 0.001 ft. too high; this error increases as the square of the distance. It is wholly eliminated if the instrument is always exactly half-way between the turning points. Refraction is not always the same; its mean effect was used in finding the above correction. Consequently, if there is abnormal refraction or a change of refraction between sights, or a tremulousness or "boiling" of the air, small errors may result. The only remedy is to shorten the length of sight, or wait for better atmospheric conditions. The atmosphere is

usually in the best condition for seeing just before sunrise and a little while before sunset, although the refraction is then greater. A cloudy day is better than a clear one.

There is a possibility of error in recording the observations and making the computations, but in precise leveling there are so many checks that there is no probability for serious error in this respect.

4. *Personal Errors.*—The errors previously described are liable to occur with any observer; they are due chiefly to the instruments and to the nature of the work, and would probably not materially differ for equally skilled observers. We come now to a class of errors which depend mainly upon inaccuracies peculiar to the individual.

With a target rod, errors of one foot and one-tenth are not uncommon. The only check is for the rodman and observer to read it independently and compare notes; but this is inconvenient and not always possible. With a self-reading rod this error is less liable to occur, especially if three hairs are read.

Finally, each individual has errors peculiar to himself, or to the work he is doing. One may read a target higher or lower than another of equal skill; or one observer in reading the position of the bubble may have peculiar views as to what constitutes the end of the bubble, or he may habitually read the bubble so as to get a distorted view of it through the glass tube; errors from these causes are compensating. Again, the target may be better illuminated on foresights than backsights, as in working toward or from the sun; in this case the error will be cumulative.

With skillful observers, all such errors are quite small and generally cancel themselves. In fact, the errors here classed as personal are possible rather than demonstrated as actually occurring; and yet, there is nothing more certain than that in any series of accurate observations, there is a difference between the results of different individuals. This difference is known as "personal equation." In long lines of accurate leveling, it has been

found that each man's way of performing each operation has a decided effect upon the final result.

§ 98. It is a curious fact, but one abundantly verified, that when lines are duplicated in opposite directions, the discrepancies tend to one sign and increase with the distance. This subject has been much discussed and various reasons assigned, as settling of instrument, settling of turning point, dis-leveling effect of the sun, illumination of the target, and personal bias in reading the target or bubble (but these should cancel in back and foresights), but none of the reasons are entirely satisfactory. "These discrepancies vary with different observers and are not even constant for the same observer, are nearly proportional to the distance, and seem to be independent of the nature of the ground, the direction in which the work is done, the season, or the manner of supporting the rod." *

The accuracy is increased by leveling alternate sections in the opposite direc-

* Chief Engineer's Report, U. S. A., 1884, p. 2555.

tion, as is done in India. It may be still further increased by reading the backsight first at each alternate time the instrument is set up.

The effect of this class of errors may be eliminated by each observer duplicating this work in the opposite direction under as nearly the same conditions as possible.

§ 99. *Limits of Precision.*—The probable error per unit of distance is generally adopted as a convenient measure of the precision reached. According to the theory of probabilities, the final error of a series of observations, affected only by accidental errors, will vary as the square root of the number of observations; hence the error of leveling a number of units of distance is assumed to vary as the square root of the distance.

This assumption would be true if accidental errors were the only ones made, and if the number of observations were strictly proportional to the distance leveled, *i.e.*, if the length of sight were constant; but in the preceding article it

was shown that leveling is affected by an error which is nearly proportional to the distance. It has frequently been noticed that, considered individually, the errors of a number of short lines were well within the limits which were prescribed to vary as the square of the distance, yet when the sum for several lines were considered the total discrepancy would exceed the limit. In other words, for a line leveled in only one direction the error is not strictly proportional to the square root of the distance. One part of the error is proportional to the square root of the distance and another portion varies nearly as the distance. Hence the shorter the distance, the easier to attain a limit prescribed to vary as the square root of the distance.

"According to the Geodetic Association of Europe, levels of precision executed of late years in Europe, show that the probable error of a line of levels of precision should never exceed

5 mm.$\sqrt{\text{distance in kilometers.}}$

(0.021 ft. $\sqrt{\text{miles}}$),

that 3 mm.$\sqrt{\text{distance in kilometer}}$ is tolerable, 2 mm.$\sqrt{\text{dist. in km.}}$ is a fair average, and 1 mm.$\sqrt{\text{dist. per km.}}$ is high precision." * The Coast Survey requires 5 mm.$\sqrt{\text{2 dist. in km.}}$ (0.030 ft.$\sqrt{\text{miles}}$). Of late years the Coast Survey's and Mississippi River Survey's work are considerable within the limit of 2 mm.$\sqrt{\text{kilom.}}$ The Mississippi River Commission's limit is 5$\sqrt{\text{kilom.}}$ (0.021 ft. $\sqrt{\text{miles}}$). "The limit on the British Ordnance Survey is 0.01 ft. per mile."

Results of leveling are often given of apparently greater accuracy than the limit above; but regularity of result and evenness of error is of more importance than occasionally small disagreement. It is usually the latter that is recorded.

If the error was determined by duplicating the work in the same direction, and especially if at the same time, as by methods III, IV, or V, the difference will

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* U. S. C. S. Report, 1882, p. 522.

be the apparent error, and necessarily be too small. The result obtained by the adjustment of a net of lines by the method of least squares affords the best method of arriving at the degree of precision.

§ 106. *Speed.*—The amount of work that an observer should do in a day can not be stated definitely; it depends upon the accuracy to be obtained, the power and delicacy of the instrument, the method pursued, the ground, and very largely upon the weather. For the very best work, even in clear weather, not more than 3 or 4 hours can be utilized. The average daily run for several seasons on the Mississippi River, using a Kern level and method II (§ 93), was a trifle over a mile per day for the entire season, and a little over a mile and a half for the days on which work was done.* On the Lake Survey, with the same instrument and method, the distance was about two miles per day for the days on which leveling was done.

* Chief Engineer's Report, 1884, p. 2462.

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